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The Main Challenges that a Teacher-in-Transition Faces When Teaching a High School Geometry Class

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THE MAIN CHALLENGES THAT A TEACHER-IN-TRANSITION FACES WHEN TEACHING A HIGH SCHOOL GEOMETRY CLASS

by

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ABSTRACT

THE MAIN CHALLENGES THAT A TEACHER-IN-TRANSITION FACES WHEN TEACHING A HIGH SCHOOL GEOMETRY CLASS

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During a semester-long action research study, the author attempted to implement a standards-based approach to teaching mathematics in a high school geometry class. Having previously taught according to a more traditional manner, there were many challenges involved as he made this transition. Some of the challenges were related to Geometry and others were related to the standards-based approach in general. The main challenges that the author encountered are identified and discussed. A plan of action for possible solutions to these challenges is then described.

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Chapter 1 – Introduction

There is currently a debate in the United States about how mathematics should be taught. The common approach to teaching mathematics is known as the traditional approach. This approach is familiar to most teachers because they experienced it as students when they attended school. It emphasizes memorizing rules and procedures (Stigler & Hiebert, 1999).

However, a new reform approach to teaching mathematics has emerged known as the standards-based approach. In 1989, the National Council of Teachers of Mathematics (NCTM) published *Curriculum and Evaluation Standards for School Mathematics,* which outlined standards for mathematics education. These standards have guided the reform. Additional publications have provided more insight into these standards (NCTM, 1991; NCTM, 1995; NCTM, 2000). This standards-based approach to teaching is different from the traditional approach and is not understood by many teachers. Its focus is to help students make sense of and understand mathematics beyond just memorizing rules and procedures (Hiebert et al., 1997; NCTM, 2000).

Researchers in mathematics education along with NCTM recommend that teachers change from the traditional approach to include more aspects of a reform approach (Hiebert et al., 1997; NCTM, 2000), but change is seldom easy. Teachers need to understand elements of a reform approach and how it differs from the traditional approach if they are going to consider making a change. They also need to know how to be successful when teaching according to a standards-based approach.

As mentioned previously, change is difficult. Even with an understanding of a reform approach and how it compares to the traditional approach, it is still hard to put it into action. Part of the reason for this difficulty is the gap that exists between educational theory and practice (Hensen, 1996; Patterson & Shannon, 1993). Understanding the theory is one important element of making a change, but successfully putting that theory into practice is perhaps the more difficult element. One approach that can help teachers to bridge the gap is known as action research (Hensen, 1996; Johnson, 2002).

During my first three years as a teacher, I taught high school mathematics in Virginia. My approach to teaching was very similar to the traditional approach. But during this time I began to question my approach and decided that I needed some time away from the classroom to think about how I wanted to teach mathematics. I decided to go back to school at Brigham Young University to pursue a Masters Degree in hopes that I could become a better teacher and feel more confident in my teaching.

During my Masters program, I had many opportunities to think about a standards-based approach and how it compared to the traditional approach. I really enjoyed aspects of the standards-based approach and wanted to try it for myself. I felt like it would be a great improvement over the traditional approach that I had been using. We spent time in my classes discussing the pros and cons of each approach and had looked at specific examples and case studies. But even with all of the background information and my desire to change, I worried that I would not be able to do it successfully. I still had too many questions about the specifics of a standards-based classroom.

This situation led into an action research thesis. I decided to teach full-time during the second year of my program while I finished my remaining classes at night. I was determined to try a standards-based approach while I was still a student. This would give me the opportunity to discuss my experiences with my professors and get feedback and advice. It would also give me a needed support system.

In order to simplify my study, I decided to focus on just one content area. I chose Geometry. During the summer prior to teaching, I spent time thinking about the important ideas of Geometry and planning lessons that would align with a standards-based approach to teaching.

This thesis describes my journey as I implemented a standards-based approach to teaching for the first time in my high school Geometry classes. It discusses the challenges I faced and the ways that I dealt with these challenges. Because action research is a cyclical process of trying something and then reflecting on it in order to improve, I propose a plan of action at the end of this thesis. The plan of action identifies things that I will try to do to improve my teaching in future years.

Chapter 2 – Literature Review

In order for teachers to transition from a traditional approach to more of a standards-based approach, they need to understand the elements of both approaches. They need to recognize how they are different and also recognize ways in which they can feel successful using a standards-based approach. Understanding these two approaches to teaching mathematics will set the stage for this thesis.

Traditional Approach to Teaching

The most common approach to teaching mathematics in the United States is known as the traditional approach. In 1978, the National Science Foundation (NSF) provided funds to researchers to better understand how teachers taught mathematics. Welch (1978) provided the following description of the mathematics classrooms that he observed:

> In all math classes that I visited, the sequence of activities was the same. First, answers were given for the previous day's assignment. The more difficult problems were worked on by the teacher or the students at the chalkboard. A brief explanation, sometimes none at all, was given of the new material, and the problems assigned for the next day. The remainder of the class was devoted to working on homework while the teacher moved around the room answering questions. The most noticeable thing about math classes was the repetition of this routine. (p. 6)

Fey (1979) summarized the findings from all of the NSF-funded research in 1978 by saying that the typical classroom approach included “extensive teacher-directed explanation and questioning followed by student seatwork on paper-and-pencil assignments” (p. 494).

Almost 20 years later, researchers found that teachers in the United States continued to teach mathematics using the same approach. The results of the Third International Mathematics and Science Study (TIMSS) included the following description:

> The typical eighth-grade mathematics lesson in the U.S. is organized around two phases: an acquisition phase and an application phase. In the acquisition phase, the teacher demonstrates or leads a discussion on how to solve a sample problem. The aim is to clarify the steps in the procedure so that students will be able to execute the same procedure on their own. In the application phase, students practice using the procedure by solving problems similar to the sample problem. (Stigler & Hiebert, 1997, p. 18).

The quotes by Welch (1978) and Stigler & Hiebert (1997) provide good descriptions of what the traditional classroom looks like on a daily basis.

The traditional approach to teaching mathematics is well-known by teachers. Many experienced the same approach as students and quickly learned that memorizing rules and procedures was the key to success. As teachers, many continue to emphasize memorization. They present new rules and procedures each class, provide examples of how to use them on sample problems, and then assign homework. The purpose of the homework is to give students the opportunity to practice using the rules and procedures on problems that are similar to the ones worked in class (Hiebert et al., 1997).

Given this background of a traditional classroom and its emphasis on memorization of rules and procedures, it is important to understand what teachers do to feel successful. These feelings of success in the classroom are commonly referred to as teacher efficacy. Certain teacher actions lead to student success which in turn leads to teachers feeling successful.

Teacher Efficacy

In a traditional classroom, actions associated with the telling model (Smith, 1996) lead to feelings of success for many students and teachers. The telling model rests on the idea that transmission of knowledge is the most effective way to teach (Smith, 1996). The main goal of teachers is to present new mathematical rules and procedures in a clear manner so that students can use them as quickly as possible. Some of the big ideas associated with the telling model which generate positive teacher efficacy are described below.

First, teachers are able to feel confident in their knowledge of the material. Each mathematics class contains certain facts and procedures for students to learn. These facts and procedures are all located in the textbook and explanations are also included. If teachers do not know how to teach a certain fact or procedure, they can refer to the textbook. Once teachers have mastered the material for the day, they can go into class feeling confident in their ability to tell the students the ideas and also to answer questions.

Second, teachers know exactly what they are expected to do. They must tell their students about the new mathematical ideas and then assign homework for the students to work on. Their most important job is preparing a lesson that introduces the new ideas to the students and prepares them to be successful on the homework assignment. If they do a good job of explaining and provide plenty of examples just like the problems in the homework, the students will be successful on the homework. When the students are performing well on the homework, the teacher feels successful.

Third, teachers know what students are supposed to do and can easily check to see that they are doing it. The telling model clearly defines the students' responsibilities. They are to "listen, watch, and practice" (Smith, 1996, p. 392). If students are listening and watching during the lesson, teachers feel successful. If students are practicing the procedures by working on homework problems after the lesson is over, teachers once again feel successful. If students do well on class tests and standardized tests, teachers feel even more successful. Once again, the goal of the traditional approach is to help students become proficient with rules and procedures.

In the traditional classroom, teachers can measure their efficacy using these three ideas. As they master the content for their particular class, perfect their lessons so that students will be ready for the homework and manage students so that they fulfill their responsibilities and perform well on tests, teachers feel successful.

Reform Approach to Teaching

The current reform began in the 1980s and has been directed for the most part by NCTM. As mentioned in Chapter 1, NCTM (1989) published the *Curriculum and*

Evaluation Standards for School Mathematics which provided a vision for teaching students both mathematical skills and mathematical understanding. The document included eleven standards—seven content standards and four process standards. In this thesis, only the process standards will be discussed. The four process standards identified were problem solving, communication, reasoning, and mathematical connections.

After eleven years, NCTM (2000) published *Principles and Standards for School Mathematics,* which included updates and revisions to the standards. The total number of standards had changed as well as the number of each type. Instead of eleven standards, there were only ten—five content standards and five process standards. The process standards had been slightly renamed and also included a new standard. These five standards were now problem solving, reasoning & proof, communication, connections and representation.

Process Standards

The first process standard is problem solving. NCTM (2000) recommended that students increasingly use problem-solving approaches to learn mathematical content. Many researchers agree that problem solving is one of the best approaches to help students make sense of mathematics (Grouws, 2003; Hiebert et al., 1996; Hiebert et al., 1997; Lester et al., 1994; Mogetta, Olivero & Jones, 1999; Taplin, 2006; Zebath, 2003). Problems can be used to introduce new material or new ideas and as the students work to solve the problems, they will gain insight into mathematics. They will also gain confidence in their ability to solve problems for which they do not have a readily-available solution method (NCTM, 2000).

The second process standard is reasoning and proof. Students need opportunities to develop their logical reasoning skills in every mathematics course, not just in Geometry where these opportunities are most prevalent. Making use of inductive reasoning and deductive reasoning gives students the opportunities to make conjectures based on patterns they have identified and then to verify their conjectures. Students should come to appreciate both forms of reasoning and recognize that deductive reasoning is the means by which mathematical ideas are verified (NCTM, 2000).

The third process standard is communication. "All students need extensive experience listening to, reading about, writing about, speaking about, reflecting on, and demonstrating mathematical ideas" (NCTM, 1989, p. 140). As students actively participate in solving problems, either in groups or as a class, they will have opportunities to discuss mathematical ideas, communicate their ideas to others, and listen to other ideas and approaches. Students need to have opportunities to communicate in the mathematics classroom if they are to become proficient in mathematics.

The fourth standard is connections. Students' mathematical understanding deepens as they have opportunities to make connections between mathematical representations, between mathematical topics, and even between mathematics and other disciplines (NCTM, 2000). Making connections helps students to recognize that mathematics consists of more than just memorizing isolated, unrelated rules and procedures.

The fifth and last process standard is representation. Representation refers to the ways in which students represent ideas. These representations can include symbols, tables, graphs or pictures. They allow students to organize information, communicate mathematical ideas and solve problems. As students improve in their abilities to use and understand different representations, they "have a set of tools that significantly expand their capacity to think mathematically" (NCTM, 2000, p. 67).

These five process standards encourage aspects of a classroom that do not typically exist in most traditional classrooms. As students engage in problem solving, they assume more ownership of the mathematics. By reasoning and communicating together about the problems, they gain a deeper understanding of the mathematics and make connections between ideas and representations. The process standards describe a classroom where student learning and student understanding can effectively take place. In contrast, most traditional classrooms limit opportunities to solve open-ended problems, as well as opportunities to reason, communicate and make connections.

With the process standards in place, NCTM needed to help teachers understand how to use them in the classroom. Two years after publishing the original *Standards* document in 1989, NCTM (1991) published the *Professional Standards for Teaching Mathematics*. This latter document supports the former by outlining for teachers the reform standards for teaching mathematics in light of the process standards.

Professional Standards

The reform standards for teaching mathematics are organized into four main categories and include six standards as follows (NCTM, 1991):

Tasks

1) Worthwhile Mathematical Tasks

Discourse

2) Teacher's Role in Discourse

3) Students' Role in Discourse
4) Tools for Enhancing Discourse

Environment

5) Learning Environment

Analysis

6) Analysis of Teaching and Learning

These six standards include elements of the process standards described previously and provide the framework for traditional teachers to understand how they can include reform elements in their classroom. Unless they understand these standards, they will not know how to make changes to their teaching. Each of the six reform standards are discussed below.

Worthwhile Mathematical Tasks. The first standard for teaching mathematics is to incorporate worthwhile mathematical tasks in the classroom. This standard fits nicely with the problem solving process standard. In order for students to develop their problem-solving skills, they need worthwhile tasks to work on and think about. NCTM (1991) identified three important areas to consider when deciding which tasks to use: mathematical content, the students, and the ways that students learn mathematics.

The first important area to consider when choosing tasks is the mathematical content. Does the task appropriately allow students to think about the important mathematical ideas for the day? Does the task prepare students for discussion of future topics and also make connections to previous topics? Does the task help students conceptually understand the mathematics and also promote procedural fluency (Hiebert et al., 1997)? A good task will focus on the important mathematical ideas, will make connections to other topics, and will help students with their conceptual understanding and their procedural fluency.

Second, students need to be considered when choosing tasks. Teachers need to be aware of their students' background and identify what they can do mathematically and what they need to learn mathematically. Students' interests are also important when selecting tasks. If students are interested in the problem, they will be more engaged in trying to find a solution. It is also important to have a sense for how far the students are willing to stretch themselves intellectually. If the problem stretches them too far, they will give up. However, if the problem is interesting to them and stretches them just right, they can make significant progress.

Third, teachers need to use their knowledge about how students learn mathematics when selecting tasks. This knowledge typically comes from both research and experience. For example, starting with concrete examples in the task before discussing abstract ideas is important. By thinking about concrete examples and making representations to understand the examples, students are in a better position to extend their thinking to more abstract examples.

In order to provide students with worthwhile tasks, teachers need to consider the mathematical content of the task, the background of their students, and the ways in which students learn mathematics. If a teacher consistently thinks about these three areas, the tasks should be engaging and should help students learn and understand mathematics.

Teacher's Role in Discourse. The second standard for teaching mathematics is to understand the teacher's role in discourse. "Rather than serve as the ultimate authority and dispenser of knowledge, the teacher variously plays the roles of guide, coach, question asker, and co-solver of problems (sometimes all these roles at the same time)" (Lester et al., 1994, p. 154). These are very important roles as the students work through problems and develop methods that make sense to them. As a guide or coach, the teacher has to decide how to orchestrate the classroom as a team in order to enhance student learning and understanding. Asking questions and acting as a co-solver are elements of this orchestration. NCTM (1991) identifies three important features of the teacher's role in orchestrating the discourse.

The first feature of the teacher's role is to elicit student reasoning about the mathematics. This can be accomplished by providing worthwhile tasks and asking good questions during the tasks. One of the best questions a teacher can ask is "Why?" because it pushes students toward an explanation of their answers or their thinking.

The second feature is recognizing that the teacher will be active in a reform classroom, but in a different way than the traditional classroom. In a reform classroom, teachers are actively listening and responding to students rather than actively talking and lecturing. As the teacher listens to student ideas, some will be more useful than others. The teacher must constantly be making decisions about which ideas to pursue and which ideas to leave behind. As a co-solver of problems, the teacher must also contribute to the discussion when needed. Sometimes this involves asking questions and sometimes it involves providing information.

The third feature of orchestrating the discourse is to manage student participation. Some students will be more willing to participate than others, but the teacher must try to get every student involved in some way. They must manage how students take turns sharing ideas and at times call on students who do not have their hand up. Teachers must also remember that participation does not have to come in a whole-class setting. Some students may feel more comfortable in smaller groups and participate more there. As teachers manage the participation, they are encouraging all of their students to make sense of the mathematics.

The teacher's role in a reform classroom is different from their role in a traditional classroom. The teacher guides and coaches the students as they explore their ideas and reasoning and provides insight when needed. The teacher asks questions to elicit students' thinking and to encourage them to reason. The teacher also focuses on giving every student the opportunity to participate and engage in the classroom.

Students' Role in Discourse. The third standard for teaching mathematics is to understand the students' role in discourse. In a traditional classroom, students are expected to quietly listen to the lecture and take notes. However, in a reform classroom students are expected to "1) explain and justify their solutions, and 2) listen to others, learn from others" (Hiebert et al., 1997, p. 16). Explaining and justifying solutions becomes a daily routine as students work on new problems. They are actively engaged in the learning process and work to find solutions that make sense to them. Listening and learning from others occurs as students present their solutions in small groups or in front of the class. As presentations are made, students must decide if they agree with the explanation, if they would add anything to the presentation or if they have found a mathematical error in the presentation. As students work together to present ideas and listen to ideas, they will be able to make sense of the mathematics.

Tools for Enhancing Discourse. The fourth standard for teaching mathematics includes tools that enhance the discourse in the classroom. Teachers need to incorporate a variety of tools in the classroom for students to use as they work to solve problems and discuss solutions. Some of these tools include technology such as calculators or computers. Other tools include using representations such as pictures, tables or graphs, all of which help to organize information. In addition to these tools, teachers can also enhance discourse by telling stories to introduce new problems. By adding context to the mathematics, students have more to discuss.

Learning Environment. The fifth standard is the learning environment in the classroom. Some of the key elements that create a positive learning environment are "a genuine respect for others' ideas, a valuing of reason and sense-making, pacing and timing that allow students to puzzle and to think, and the forging of a social and intellectual community" (NCTM, 1991, p. 6). Teachers are central in developing this environment. They send messages daily about what is important and what isn't important. Teachers have to dictate the expectations through their own actions. For example, they can teach students to respect the ideas of classmates by respectfully considering student ideas during class. As a second example, they can teach students that reasoning is important by consistently asking students for their explanations. Creating the right kind of learning environment is very important when teaching mathematics according to a reform approach. Students have to feel comfortable in the classroom if they are going to try alternative approaches and contribute their thinking.

Analysis of Teaching and Learning. The last of the six professional standards involves the analysis of teaching and learning in the classroom. As teachers work to help students make sense of mathematics, they must step back from time to time to evaluate what is happening in the classroom. They need to identify positive results from their attempts to include more elements of a reform approach and they also need to identify challenges they are facing in order to fix them. The solutions may require extensive thinking, research and collaboration, but the result is an improved learning environment for the students.

The six standards for teaching mathematics as defined by NCTM (1991) identify the areas that teachers need to work on when attempting to include more of a reform approach in their teaching. Worthwhile tasks, the teacher's role, the students' role, enhancing discourse, the learning environment and analyzing the teaching and learning are important elements of creating a classroom where students can understand and make sense of mathematics. With these new elements in the classroom, how will teachers know if they are influencing students in a positive way?

Teacher Efficacy

Teacher efficacy as it relates to a reform approach is different from teacher efficacy in a traditional classroom. Because teachers have new roles in a reform approach, they must redefine how they build their efficacy. Smith (1996) suggests four moorings that may provide teachers a way of measuring efficacy in their new role—1) choosing problems, 2) predicting student reasoning, 3) generating and directing discourse and 4) judicious telling. Each of these moorings is related to the others, but each provides a different sense of efficacy.

Choosing Problems. Choosing problems is one way to measure efficacy. If teachers choose motivating problems, they will immediately see the fruits of their labors. The students will quickly engage in the problem and significant learning will occur. If teachers choose problems that are not motivating to the students or perhaps too difficult for the students, they will not see much learning. The students will not make much progress.

Predicting Student Reasoning. Predicting student reasoning also helps teachers to measure efficacy. Since the focus of a reform curriculum is on student thinking, teachers should be able to make some predictions about how their students will think about a new problem. They should get better throughout the year as they get to know their students better. “Even partially accurate anticipation of the character of students’ reasoning provides a sense of stability and a basis for managing classroom discourse” (Smith, 1996, p. 397).

These first two moorings of choosing problems and predicting student reasoning follow from the professional standard of implementing worthwhile tasks. The teacher must identify and use engaging tasks but must also consider how students might reason about the task. Predicting the student reasoning will influence the questions that are asked during the task and the overall structure of the task.

Generating and Directing Discourse. Generating and directing discourse follows from the previous two moorings. After choosing a motivating problem and anticipating student responses, teachers must be able to generate and direct a good discussion. One objective of discourse is to involve the students and encourage them to share their ideas. Another objective is to use these ideas to understand a problem more deeply. When working alone, students often see a problem in only one way. As they engage in discourse, they “can increase their understanding of the problem and of different methods that work” (Hiebert et al., 1997).

Teachers have to carefully consider when to ask questions or make comments and when to let students discuss their ideas. If teachers jump in too often or too soon to resolve a disagreement about the mathematics, they may be viewed by the students as the sole authority. This will take away from the students' willingness to share their own ideas or to continue thinking about the problem.

This third mooring follows from two of the professional standards—teacher's role in discourse and learning environment (NCTM, 1991). The teacher is responsible for guiding and coaching the students through the mathematics involved in the tasks and they are also responsible for creating a learning environment where students feel comfortable contributing to the classroom discourse. If they have created the right type of environment and are skilled at guiding the discussion, they will feel effective.

Judicious Telling. Judicious telling is closely related to generating and directing discourse. At times, teachers need to do a little telling to help students understand the problem or to help them think about possible approaches. However, teachers need to be very careful not to lower the mathematical level of the problem by telling too much (Hiebert et al., 1997; Stein, Smith, Henningsen & Silver, 2000). Telling can also be helpful to bridge the gap between student thinking and accepted mathematical notation.

This mooring of judicious telling also relates to the teacher's role in discourse. Guiding and coaching students through the tasks is done in different ways. Sometimes it involves asking questions to encourage students to keep thinking about a problem. But at other times, it involves telling students information such as new mathematical notation. Teachers certainly need to tell students information at times, but should be judicious in their telling.

As teachers attempt to change their approach to teaching mathematics, these four moorings will assist them in helping them feel successful. They can focus on planning good tasks that will engage the students as well as on creating an environment that will allow for good discourse in the classroom. Teachers can feel successful in a reform classroom by striving to master these elements.

Action Research

As mentioned in Chapter 1, action research is an approach that teachers can use to help them bridge the gap between theory and practice. Action research is a process of studying a school problem or situation given a theoretical background. The goal of action research is to improve the problem or situation by combining insight gained from being in the environment as well as insight gained from theory. "It is a systematic and orderly way for teachers to observe their practice or to explore a problem and a possible course of action" (Johnson, 2002, p. 13).

Action research provides the perfect framework for teachers who desire to change their teaching approach. After researching the theoretical standards and goals of a reform approach, they can try to apply these ideas into their own classroom and observe the results. Based on the results, they make changes. This thesis describes a large cycle of the action research process that lasted a semester but there were many smaller cycles along the way as illustrated by the following example.

One struggle that students had during this study was understanding similarity. After a couple of tasks, it became apparent that they did not understand scale factor. Before continuing with more tasks, time was taken to discuss the topic with Dr. Peterson and determine new tasks that might help the students. These tasks were then implemented a week later and the students had a little more success than they did with the first tasks. This briefly illustrates the nature of action research. The teacher tries something, evaluates it, looks for resources and then tries something else.

Once action research has been selected as the framework to conduct the study, a content area might also be selected. Since the action research study for this thesis will be conducted at the high school level, the possible content areas include Algebra 1, Geometry, Algebra 2, Pre-Calculus and Calculus. However, the author was only teaching Algebra 1 and Geometry that particular year and decided to look at Geometry for the study. In order to implement reform standards in a Geometry course, it is important to understand the standards, but it also helpful to understand specific research pertaining to Geometry.

Geometry

Geometry differs from other mathematics courses in its content as well as its focus. The content of Geometry includes the study of shapes such as triangles, quadrilaterals and circles. Studying shapes provides a visual, concrete mathematics course as opposed to a course such as Algebra, which is mostly abstract and symbolic. One of the focuses of Geometry is to use these shapes as a means to develop reasoning and justification skills (Fawcett, 1938). Developing these skills is one of the process standards of the reform. This is accomplished as students learn to prove that certain ideas and results are true.

As mentioned above, one of the focuses in Geometry is the development of reasoning and justification skills.

> "…[T]he reason we teach demonstrative Geometry in our high schools today is to give pupils certain ideas about the nature of proof. The great majority of teachers of Geometry hold this same point of view. Some teachers may at first think our purpose in teaching Geometry is to acquaint pupils with a certain body of geometric facts or theorems, or with the applications of these theorems in everyday life, but on second reflection they will probably agree that our great purpose in teaching Geometry is to show pupils how facts are proved" (Upton, 1930, pp. 131-132).

NCTM (2000) stated more recently that "Geometry is a natural place for the development of students' reasoning and justification skills, culminating in work with proof in the secondary grades" (NCTM, 2000, p. 41). These two statements were made seventy years apart, yet both emphasize the importance of using Geometry as a means to teach students about mathematic proof. NCTM (2000) clarifies that proof is not the only reason for teaching Geometry; geometric modeling, spatial visualization and spatial reasoning are other important focuses in Geometry.

Proof

Proof is an important part of Geometry but it is not expected that the students involved in this study will master it by the end of the year; it is a long-term goal. However, proof will be discussed over the next several pages because much of what is done in Geometry lays the foundation for proof. Understanding why students struggle with proofs, understanding why they are unable to master it in one year, and understanding how they can improve their ability to do proofs are all addressed.

First, proof will be defined.. A typical definition of proof is: “A careful sequence of steps with each step following logically from an assumed or previously proved statement and from previous steps” (NCTM, 1989, p. 144). Often, a person might recall 2-column proofs, which involves making statements in one column and providing the justification for those statements in the second column. Two-column proofs are one type of proof used mainly to verify or establish the validity of a conclusion.

In addition to using proof to establish validity, de Villiers (1999) identified five additional functions of proof—proof as explanation, discovery, systematization, communication, and intellectual challenge. It is not the form of the proof (e.g. 2-column proofs) that matters, but rather the thinking involved and the ways of developing that thinking. “The functions of proof that may have the most promise for mathematics education are those of explanation and communication” (Yackel & Hanna, 2003, p. 228). Notice that Yackel and Hanna focus on the functions of proof that help students develop thinking skills rather than on the form that students use to organize their thinking.

Why Students Struggle to Master Proof

Historically, many students have struggled to understand and master proofs. Usiskin (1982) found that only 50% of high school graduates had taken Geometry. Senk (1985) conducted a study to determine how well Geometry students were able to write proofs. She revealed that only 30% of the 2700 Geometry students she studied had mastered proofs. Together, these two studies inform us that in the mid 80s only 15% of our high school graduates had mastered proofs. McGivney and DeFranco (1995) believe that “these alarming statistics may be a direct result of the instructional strategies used to teach Geometry, namely the memorization of facts and theorems” (McGivney & DeFranco, 1995, p. 552). This problem may be a result of the traditional approach to teaching mathematics, but includes more than just that.

Research has revealed a few ideas that help us understand why students struggle with proof. First, the type of thinking necessary to complete proofs is different from day-to-day thinking (Yackel & Hanna, 2003). This is not surprising. Students have not been exposed to proofs prior to Geometry, but are immediately expected to think and reason formally. Teachers can help students make this adjustment by building on their day-to-day thinking and helping them to develop the skills necessary to reason formally.

Second, students have learned to rely on teachers to validate their thinking and never acquire the ability to think for themselves. Instead of understanding the mathematical relationships well enough to know that an answer is correct, most students ask the teacher if they did it right or check the answers in the back of the book. It is becoming clear that “as long as students rely on the teacher to decide on the validity of a mathematical outcome of their activity, the word ‘proof’ will not make sense for them as we expect it to do” (Balacheff, 1991, p. 179). Students have to understand the basics of mathematics and make sense of the relationships in order to be successful with proofs.

Third, most students are not at a level of Geometric thinking that allows them to do proofs. To understand levels of Geometric thinking, we must look back almost 50 years ago.

The van Hiele Theory. In 1959, Pierre van Hiele and his wife Dieke van

Hiele-Geldof worked to understand how students thought about Geometry and proposed a theory of five different Geometric levels of thinking, known as the van Hiele levels. Over the years, the van Hiele theory has been tested repeatedly and is now widely accepted by the mathematics education community as a good measure of Geometric thinking. Their paper discussing the theory was originally published in Dutch, but gained attention in the United States in the late 1970s when it was translated to English. Burger and Shaughnessy's (1986) descriptions of the van Hiele levels are provided in Table 1.

Previously, it was mentioned that students are not at a level of Geometric understanding to write proofs. With the van Hiele levels as a guide, we can analyze this statement now. In order to master proof writing, a student needs to be thinking at approximately level 3. Unfortunately, students rarely reach level 3 by the end of a full-year Geometry course. This is because over 70% of students enter at a level 0 or 1 (Usiskin, 1982) and cannot develop their thinking skills enough in one year to reach a level 3. In order to master proof writing, students should enter a Geometry class at a level 2 or higher (Senk, 1989; Battista & Clements, 1995). If students do not have the opportunity to develop their Geometric thinking to a level 2 or higher prior to entering Geometry, they are usually not prepared to think about proofs and resort to memorization as the only way they can complete proofs (Burger & Shaughnessy, 1986; Senk, 1989). The three reasons listed above help us better understand why students struggle to master proofs. Before discussing a possible solution, it is also important to understand how students move from one van Hiele level to the next.

Table 1

Description of the van Hiele Levels (Burger & Shaughnessy, 1986, p. 31)

Levels	*Description*
0	(Visualization) The student reasons about basic geometric concepts, such as simple shapes, primarily by means of visual considerations of the concept as a whole without explicit regard to properties of its components.
1	(Analysis) The student reasons about geometric concepts by means of an informal analysis of component parts and attributes. Necessary properties of the concept are established.
2	(Abstraction) The student logically orders the properties of concepts, forms abstract definitions, and can distinguish between the necessity and sufficiency of a set of properties in determining a concept.
3	(Deduction) The student reasons formally within the context of a mathematical system, complete with undefined terms, axioms, an underlying logical system, definitions, and theorems.
4	(Rigor) The student can compare systems based on different axioms and can study various geometries in the absence of concrete models.

Phases of the van Hiele Theory. In order to progress from one van Hiele level to the next, a student needs to be guided through five different phases (Fuys, Geddes & Tischner, 1988). These five phases are identified and described in Table 2. Students must pass through each of these five phases in order to move from a van Hiele level 0 to a level 1 and then again to move from a van Hiele level 1 to a level 2. In order to help students reach a level 3, the instruction must be designed to give students the opportunities to move through the phases.

Table 2

Five Phases of the van Hiele Theory (Fuys, Geddes, & Tischner, 1988, p. 7)

Phases	*Description*
Information	The student gets acquainted with the working domain (e.g., examines examples and non-examples).
Guided Orientation	The student does tasks involving different relations of the network that is to be formed (e.g., folding, measuring, looking for symmetry).
Explicitation	The student becomes conscious of the relations, tries to express them in words, and learns technical language which accompanies the subject matter (e.g., expresses ideas about properties of figures).
Free Orientation	The student learns, by doing more complex tasks, to find his/her own way in the network of relations (e.g., knowing properties of one kind of shape, investigates these properties for a new shape, such as kites).
Integration	The student summarizes all that he/she had learned about the subject, then reflects on his/her actions and obtains an overview of the newly formed network of relations now available (e.g., properties of a figure are summarized).

Standards-Based Approach in Geometry

Because the five phases are essential in order for students to move from one van Hiele level to another, finding connections between them and the process standards of a standards-based approach is important and is what follows. In review, the five process standards are problem solving, reasoning & proof, communication, connections and representation (NCTM, 2000).

Problem solving is an important component to help students to work through the phases of the van Hiele model and ultimately master proof writing. "Geometry, especially proof writing, has to be viewed as a problem-solving activity" (McGivney & DeFranco, 1995, p. 555). Problem solving helps students in the Information phase to become "acquainted with the working domain". As students wrestle with new problems, they gain a deeper understanding of the domain. Problem solving also helps students in the Guided Orientation phase and the Free Orientation phase. Both of these phases require students to work with tasks, or problems, to better understand relationships. Problem solving is a component that is integral throughout all of the phases in providing opportunities for students to learn mathematics.

Reasoning and proof is a standard that shows up in all of the phases. Students have to reason as they work with tasks and also as they try to express themselves in words. As they try to understand new relationships, they are constantly thinking and reasoning.

Communication is also important in helping students to move through the phases, especially the Explicitation phase. This phase requires students to try to express the relationships that they have found in words. As students communicate their thinking, they practice expressing these relationships and they learn to use the technical language better by doing so.

The standard of communication is also important to consider as students learn their new roles. One of their new roles is to explain their reasoning which helps them to build towards proof. It is not expected that they will reach a van Hiele level 3 where they can actually do proofs, but they can certainly improve their ability to explain their reasoning. As mentioned earlier, proof is not the expectation for this year but rather the long-term goal. It is the students' ability to explain their reasoning and to justify their solutions that will be the focus. Whenever these ideas are discussed, the associated goal of proof is inferred. As a side note, it is important to recognize that the students' role of explaining their reasoning actually helps them build toward proof in any mathematics class, not just Geometry where it is a focus.

The standard of connections is also important in the van Hiele model. The fifth phase, known as Integration, requires students to look back at what they learned and to summarize their findings. This gives them an opportunity to make connections between the ideas they have learned.

Representation can also help students. During the tasks, students will gain a deeper understanding of the mathematics as they consider different representations. Drawing diagrams, making models and using tables are just some of ways in which students could make sense of problems and reason about possible solution strategies.

The process standards that guide a standards-based approach to teaching mathematics can provide an avenue for students to move through the van Hiele phases and ultimately the van Hiele levels. As teachers implement these standards, they should theoretically see that more students are moving from one van Hiele level to the next and moving closer to successfully learning to reason and prove. They should be able to lay the foundation for students to learn to do proof as they involve them in problem solving and the other process standards. However, as mentioned previously, there is typically a gap between theory and practice. Teachers need to conduct action research studies to observe what really happens.

Action Research Question

I have described the traditional approach and a reform approach to teaching mathematics. I have identified the importance of action research to bridge the gap between theory and practice. I have also identified Geometry as the class where I will conduct the action research. I have identified important research related to Geometry as a background for understanding the content and the levels of student thinking. I have discussed how the process standards of the reform can assist in meeting the challenges of Geometry and now desire to try all of this theory in my own classroom. Since I am in the process of transitioning from a traditional approach to more of a reform approach to teaching mathematics, I will refer to myself as a teacher-in-transition.

I anticipated that there would be situations and frustrations for me as a teacher-in-transition that would require mental and emotional energy to resolve. If I was not able to resolve these situations or frustrations quickly and they were related to my standards-based approach, I referred to them as challenges. Some of these challenges apply to a standards-based approach in any classroom whereas some apply specifically to Geometry. These challenges would most likely include aspects discussed in the literature but would probably also include aspects that did not. I anticipated that I would struggle with my efficacy in this new approach but I did not know how I would respond to my struggles. I would probably have a hard time planning worthwhile tasks at the right van Hiele level. I anticipated that it would be a challenge to direct the discourse in terms of finding a balance between letting students struggle, asking questions and judiciously telling. I also anticipated that it would be a challenge to develop an environment where students felt comfortable sharing their ideas and taking risks. I figured that there would be other challenges as well that would not appear until I started putting the ideas of a reform into practice. With that background, my question for this thesis was:

What are the main challenges that a teacher-in-transition faces when teaching a high school Geometry class?

Chapter 3—Methodology

As discussed in Chapter 2, this thesis is an action research study. Action research involves a cyclical pattern of trying to improve a situation or solve a problem in a classroom. Johnson (2002) outline five steps in the action research process. The first step is to identify a question or problem to study in the classroom. The question that I identified in Chapter 2 was: What are the main challenges that a teacher-in-transition faces when laying the foundation for students to do proof in a high school Geometry class?

The second step of action research is to decide what data to collect and how to collect it and the third step is to collect and analyze the data. I discuss what data I collected, how I collected it and how I analyzed it later in this chapter. The fourth step is to describe how the findings can be used to create a plan of action to answer the question that was posed. This is an important part of action research and is discussed in Chapters 4 and 5. The fifth step of action research is to share the findings and the plan of action with others. This thesis will be published, thus providing access for others to read the findings and the plan of action.

The five steps of action research are repeated again and again until the problem is solved. Once all five steps have been completed, the new plan of action can then be used to identify a related question and the process begins again. This thesis completes the five steps once, but future studies could use the results of this thesis to complete a similar study.

Participants

Researcher

The main participant in any action research project is myself, the researcher. So, in this project, it is important for me to reveal some of my background and beliefs that will impact what I saw and experienced. I graduated from Brigham Young University (BYU) in 2002 with a bachelor's degree in Mathematics Education. Mathematics had always been easy for me and I wanted to help others find satisfaction in this important subject.

Upon graduating from BYU, I accepted an offer to teach in Virginia as a member of Fairfax County Public Schools. I taught at two different high schools for a total of three years. My approach to teaching was very similar to the approach I had seen as a student, which was the traditional approach. However, it was a little different. Instead of just presenting material, I tried to help students to really understand how to use a procedure. I taught them songs to remember formulas and tried to make each new procedure fun in some way. This was the extent of my understanding about a mathematics reform. I had taken two mathematics education classes at BYU and had been exposed to the ideas of teaching for understanding. However, I believed that this simply meant an understanding of how to use a procedure rather than an understanding of where the procedure comes from or why we use it.

For the first two years I felt very successful in my teaching role. I measured my efficacy according to the traditional approach as described in the previous chapter. I was prepared to tell my students about the "manageable mathematical content that [I had] studied extensively" and I knew "what [I] must do with that content to affect student learning" (Smith, 1996, p. 388). The mathematical content that I needed to teach came straight from the book and each year was the same. From the first year to the second year, I had a better idea of areas where students struggled and tried to find ways to teach the ideas in a more clear and fun way. I felt like I was able to affect student learning by providing clear explanations of how to use the procedures and by making class fun and upbeat with songs and games. My students did well on my quizzes and tests and their scores on the end-of-year state test were a little above the average in the school. I felt good about the job I was doing.

During my third year of teaching I started to question my approach. One student repeatedly asked me why she needed to learn Algebra. I would joke around with her at first and give her answers like, "You will need to learn it for the final exam" or "You will need to know it to pass the end-of-level test". However, the more I thought about these answers, the more I was disappointed in them. Was there a good reason for all students to learn mathematics beyond passing some tests required by the school and the state? I struggled to find a good answer to this question, especially when I saw how much some students despised Algebra. Was it really important for all students to learn Algebra?

During this third year of teaching, I decided that I needed to spend some time away from the classroom to find answers to my question. So, after my third year of teaching, I returned to BYU to work on a Masters degree in Mathematics Education. I was determined to become a better teacher and to help my students to recognize that mathematics would be useful to them, even if they didn't major in a mathematics-related field.

The classes that I took as part of my Masters degree provided many new ideas and helped me to mold a new perspective known as a standards-based approach. This approach, which I described in Chapter 2, was very interesting but also very different from anything I had experienced. I agreed with the ideas of a standards-based approach, but feared that I would not be able to successfully implement them in my own classroom.

The research papers provided lots of information about how to teach in this manner, but I still had so many questions about what it would really look like. I knew what a traditional classroom would look like because I had already taught for three years but I struggled to think about the details of a reform classroom. What would homework be like? What would quizzes and tests look like? How would I come up with appropriate tasks? I still had so many questions that were not really addressed in the literature. The professors offered some suggestions, but not enough for me to really feel confident that I could do it myself.

During this same time, I was trying to decide what to do for my thesis. I contemplated teaching a unit in another teacher's classroom to try a standards-based approach for myself, but ultimately decided to go back to teaching full-time in order to be in my own classroom. I wanted to have the opportunity of building the social culture that would allow the students to engage in problem solving. It turns out that I was able to teach full-time in a public high school during my second year of graduate school and still complete my remaining classes. By teaching while still a graduate student, I had lots of opportunities to discuss ideas with the professors and better understand how to be successful with this new teaching style.

Most high schools in Utah are grades 10-12 and this particular high school was no different. In Virginia where the high schools are grades 9-12, most of my Algebra 1 and Geometry students were 9th graders. But at this school, I would be teaching mostly 10th graders in my Algebra 1 and Geometry classes. This population of students would be different from anything I had experienced. Almost everything about this year of teaching was new to me—new school, new faculty, new administration, new community, new parents, new students, and trying to teach in a new approach. I was positioned to learn a lot from this action research study.

Students

In the previous section, I revealed some of my background as a participant in this study. The other important participants were the students themselves. It is important to understand their community, their school and their previous mathematics experiences. These students came from a strong religious community. Most of them were actively involved in their church. They also received a lot of support from home to do well in school and earn good grades. Overall, the students that attended this high school were good students.

This high school was one of just three in the district. Over the years, it had become known as the high school that set the academic standard for the district. Two measures of this academic achievement are the Advanced Placement (AP) scores and Concurrent Enrollment classes taken. Both of these types of classes are challenging and meet the same standard as a college class. Table 3 provides data of the three high schools in the district.

Table 3

Comparison of Three High Schools in the District

School	*Enrollment*	*AP Tests Taken*	*AP Pass Rate*	*Concurrent Enrollment Credit Earned*
Our School	1559	278	67%	3046
School #2	1765	209	68%	2652
School #3	1534	147	60%	1304

The students in the school that were part of this research study were my Geometry students. I taught three classes of Geometry and had over 100 students in these classes during the first semester. However, only 87 of these students were enrolled the entire semester; some transferred in and some transferred out during the course of the semester for a variety of reasons. Of the 87 students, 76 were sophomores, 8 were juniors and 3 were seniors. In terms of race, 81 were Caucasian and 6 were Hispanic. In terms of gender, 46 were male and 41 were female.

Because the majority of my students were sophomores, I was interested in how they were placed in the school in relation to other sophomores. Table 4 shows how the 543 sophomores were placed. Placement is certainly not the best measure of mathematical ability, but it provided an indicator. For example, it is probable that the best mathematics students in the sophomore class were part of the 40% that were taking either Algebra II or Pre-Calculus.

Table 4

Placement of the Sophomores

	Pre-Calculus	*Algebra II*	*Geometry*	*Algebra 1 & Applied Math 1* [1]	*Other* [2]
Number of Students	54	165	205	89	30
Percent of Students	9.9%	30.4%	37.8%	16.4%	5.5%

[1] Applied Mathematics I contains the same concepts as the Algebra 1 course, with necessary extensions that prepare students for future technical training.

[2] This category includes students who were taking resource math, students who were not enrolled in a math class, students who were taking an independent study math class or students who were taking math via home school but were still enrolled at some classes at the high school.

Materials

Curriculum

I spent the summer after my first year of graduate school thinking about the curriculum that I wanted to teach and the overall environment that I wanted to create in my classroom the following year. I spent many hours reading research papers about a standards-based approach as well as papers about Geometry and teaching proof. The paper that influenced my thoughts the most as I thought about teaching Geometry was the paper by Burger and Shaughnessy (1986), in which they described the van Hiele levels of Geometric thinking.

As I started to put together a sequencing of topics for the beginning of the year, my main goals were to maintain a standards-based approach and to select tasks for each topic that were at an appropriate van Hiele level and that would help students move through the phases. If the van Hiele level was too high for the students at the beginning, they would not be able to make significant progress moving through the phases. If this happened, a standards-based approach would not be very effective. The research that I mentioned in my literature review indicated that most students start Geometry at either a van Hiele level 0 or 1. Knowing this information, I decided to start the year with tasks at a van Hiele level 0 and then move to a van Hiele level 1 after several weeks. I would then attempt to use tasks throughout the semester that started at the van Hiele level of the majority of the students. The tasks would be designed to give students the opportunity to move through the phases in order to progress to the next level. As I arranged topics, I kept this in mind. I needed to place topics in an order that would allow students to build on their knowledge. I collected problems from a variety of sources such as the Geometry textbook, research papers, Dr. Peterson's experience, or my own experience.

Along the way, I had to consider other factors as well. First, I had to make sure that I was including all of the topics from the Utah State Core for Geometry (Utah State Office of Education, 2002). I could jump around in an order that made sense to me, but I needed to make sure that my students had an opportunity to understand all of the big ideas from Geometry.

Second, I had to keep in mind the pacing of the other Geometry teachers and try to be in the same place by the end of the semester. In our school, every class is considered a semester class. There is a Geometry "A" class for the first semester and then a Geometry "B" class for the second semester. This approach to scheduling allows students to have one schedule for the first semester and then have a different schedule for the second semester which could include a different teacher. Thus, it was important to address the same topics in Geometry "A" as the other teachers of Geomety.

Third, I considered the recommendations for 9-12 Geometry provided by NCTM (2000). These recommendations helped me to develop ideas about the different ways to approach Geometry—hands-on constructing, using coordinates, and using transformations to view and study Geometric shapes. The time I spent thinking about these recommendations came early on in the research. They helped me develop a framework of different approaches I could use and aided me when I considered how to teach a certain topic. With all of these factors in play, I ultimately came up with an outline of topics and problems to use during Geometry "A" (see Appendix A).

Two examples of Geometry tasks that were developed are described below. The first task involved constructing triangles in an attempt to understand the differences between Side-Angle-Side (SAS) and Side-Side-Angle (SSA) congruency theorems. The second task involved using Geometer's Sketchpad (GSP) to identify properties of quadrilaterals. These tasks were designed to build on previous knowledge and to allow the students to think about some of the big ideas of mathematics.

Triangle Activity. The first activity was planned for the 2nd week of school. Triangle congruence is an important idea in Geometry and shows up throughout the year. This activity would give the students an opportunity to really understand the difference between SAS and SSA. The task was developed from an activity where students tried to construct the teacher's secret triangle given certain information about the triangle (Miller, 2006). The teacher would give the students SAS information or Angle-Angle-Angle (AAA) information to show them that they could construct the exact triangle in some cases but not in others. To make the task more open ended, the following task was developed: Construct as many different triangles as you can that have a side length of 3 inches, a side length of 4 inches and an angle of 35 degrees. As an extra incentive, a pile of candy was offered as a reward for the person or group (they would initially work individually but would have the option of forming groups later in the task) that constructed the most triangles. It was anticipated that several of the students would move the angle of 35 degrees to different locations in the triangle and understand that the order and arrangement of sides and angles would produce different triangles.

This task was developed to be accessible to every student. They would most likely know what a triangle was and they would have spent time before this activity becoming comfortable using a ruler and a protractor. A goal during the implementation of the task was that each class would collectively be able to find the four possible triangles. The task would encourage them to think about the properties of triangles and the arrangement of sides and angles. This task would also provide an opportunity for students to communicate their thinking and their reasoning as they tried to construct different triangles. This would be a nice activity to introduce SAS and SSA and build a solid foundation to understand other arrangements.

Note that this task would move through the first few phases well. Students would be given the opportunity to become familiar with different triangles by constructing them (Information). Then, they would compare their triangles to other students' triangles and a classroom discussion would ensue (Guided Orientation). Next, students would be given the opportunity to explain what makes the triangles different (Explicitation). Future tasks and activities would allow the students to continue working through the phases.

Quadrilateral Activity. I anticipated that the second activity would occur after a month of school and would involve the properties of quadrilaterals. By this time, I anticipated that the students would have explored triangles, angle pairs and parallel lines with transversals. It was assumed that most students would have had the opportunity to think at a level 1 by this time and would be able to build on their knowledge to try and think about quadrilaterals at a level 1. Level 1 involves looking at the properties of a shape and they would probably be able to look at the properties of quadrilaterals since they had spent time looking at the properties of triangles, angle pairs and parallel lines with transversals. Technology, namely GSP, would be used to aid them as they worked to identify the properties of quadrilaterals.

A file was created with several different types of quadrilaterals and access to this file was given to each student in the computer lab. Before starting them on the activity, I anticipated that they would be taught how to use GSP to measure angles and line segments. They would also be walked through an example of what was expected. They would be shown how to measure the angles, sides and diagonals of a quadrilateral and also how to identify the properties that were always true. This activity would really help them to understand the properties of quadrilaterals because of the opportunity to explore the shapes in a dynamic environment. Instead of trying to draw several quadrilaterals by hand that had the same properties, the dynamic environment would allow them to simply click-and-drag points of the quadrilateral. The shape would change, but the properties would be maintained. It was expected that this would allow them to see the properties more clearly. They would also be placed in small groups so that they would have an opportunity to communicate their reasoning to other students as they explored the shapes on their individual machines.

These two tasks provide some insight into how some of the tasks were designed and how they built on previous topics in the course. They also show how the process standards were included as worthwhile tasks were created in order to give students the opportunity to communicate, to reason and to make connections. Initially, most tasks were not planned out in this detail. Rather, some possible problems and approaches were simply sketched out with a plan to finalize the lessons once student thinking had been observed. The lessons would be modified based on how the students responded to previous tasks. If a gap in student understanding appeared during a particular lesson, the following lesson would be modified to try and address it.

Instruments

In order to measure student learning and also to get feedback on the students' perception and opinions of a standards-based approach, two additional documents were used as part of the instruments for this project. The first instrument (see Appendix B) was part of a van Hiele Geometry test developed in 1980 by The Cognitive Development and Achievement in Secondary School Geometry (CDASSG) project. The original test included a total of twenty-five Geometry questions, grouped into five groups of five questions. Each group related to a level of van Hiele thinking. If a student correctly answered at least four out of the five questions in a particular section, they were classified to be thinking at that level. For example, if a student answered four or five right in the first section, they were classified at a van Hiele level 0. If they also answered four or five right in the second section, they were classified at a van Hiele level 1. Each student was classified at the highest level that they mastered. It was anticipated that none of the students would be able to think at a level 4 so only the first twenty questions from the test were included.

Second, a survey (see Appendix C) was created for the students to take that addressed several different issues. I anticipated that their answers would provide feedback regarding their view of themselves as students and as learners of mathematics. I anticipated that it would also indicate how they viewed a standards-based approach to teaching mathematics and whether or not it helped them learn mathematics better than the traditional approach.

Procedures

Implementation of the Curriculum

The action research study began on the first day of the new school year, August 22, 2006. The study ended on the last day of the first semester, January 11, 2007. During this time, most of the pre-planned lessons and activities were implemented. Some lessons required additional time and additional activities, some lessons failed miserably and had to be approached differently, and some lessons never happened either because time ran out or because they were too advanced for the students. Some of these experiences will be discussed in more depth in Chapter 4.

Data Collection

During the semester, data was collected in many ways. The van Hiele pre-test was administered during the 2nd week of the semester and then again as a post-test during the last week of the semester. The results of that test are found in Appendix D. Four periods from each of the three Geometry classes were videotaped (twelve classes total). These classes included the activity on constructing triangles (this lasted two class periods) and the quadrilateral activity using Geometer's Sketchpad (this lasted two class periods). During the last week of the semester, the students were asked to fill out the survey. The results of this survey are found in Appendix E.

In addition to the data described above, data was also collected through the author's own observations. Seventeen journal entries were written during the semester which included the author's feelings and efficacy concerning the classes as well as specific events or comments that provided insight into student thinking.

Analysis

The seventeen journals that were written during the semester served as the primary data source. These journals were analyzed by identifying challenges. Whenever I found a situation or frustration that required mental or emotional energy to solve, I identified it as a challenge. Once a challenge was found, it was categorized according to one (or sometimes two) of the six professional standards; some challenges related to more than one of the standards. These are the six standards by which teachers can measure their progress in changing from the traditional approach to more of a reform approach. Therefore, it was important to understand some of the challenges that teachers might experience in relation to each category. It was also important to understand how these challenges affected teacher efficacy.

The challenges under each standard were then compared to each other in order to identify patterns and reoccurring themes. In order to produce valid conclusions from the data, other data sources were used to triangulate the findings in the journal entries. These sources included the van Hiele test results, the videos and the surveys.

Chapter 4 – Findings

The research question was: "What are the main challenges that a teacher-in-transition faces when teaching a high school Geometry class?" The number of times that a new or recurring challenge appeared under each standard was totaled in order to identify the standards that were the most challenging. The totals for each standard appear in Table 5.

Table 5

Total Challenges for Each Category

Professional Standard	*Total*
1. Worthwhile Mathematical Tasks	9
2. Teacher's Role in Discourse	6
3. Students' Role in Discourse	9
4. Tools for Enhancing Discourse	1
5. Learning Environment	2
6. Analysis of Teaching and Learning	0

Notice in the table that none of the challenges were categorized under the last professional standard which is Analysis of Teaching and Learning. All of the challenges could certainly be categorized under this standard, but the specific types of challenges needed to be identified in order to answer the research question. Analyzing the teaching and learning in my classroom essentially sums up the purpose of this entire thesis. The challenges needed to be specific.

It is clear from the table that the main challenges involved the standards of Worthwhile Mathematical Tasks, Teacher's Role in Discourse and Students' Role in Discourse. The findings that were coded under each of these standards are discussed in this chapter. Other data sources are included in the discussion to triangulate the findings and provide validity.

Worthwhile Mathematical Tasks

There were two main areas of selecting worthwhile tasks that were really challenging. First, it was a challenge to know how to teach students about new practices when working on tasks. Second, it was a challenge to select content at the appropriate van Hiele level.

New Practices

When working on tasks, students are often expected to "see" what teachers "see". However, they are not always familiar with the same practices. The Quadrilateral Activity provides an example of this.

I described the Quadrilateral Activity in Chapter 3 and anticipated that my students would be able to identify the properties of quadrilaterals. As planned, I demonstrated what I expected from my students before setting them loose to explore some of the quadrilaterals on their own. First, I taught them how to use GSP to measure segment lengths and angles. Then, I walked them through the process of using the measurements and the dynamic nature of the software to try to identify properties that were always true. Even with this demonstration and discussion, many students still struggled to understand what we were trying to accomplish.

The first quadrilateral that the students explored had one pair of adjacent sides that was congruent. They used the measuring tool in Geometer's Sketchpad to measure each of the sides and noticed that two of the sides were the same. However, when some of the students dynamically changed the shape to identify properties that were always true, they didn't notice anything. They said that nothing was staying the same because all of the side lengths were changing. They did not notice that two of the sides were always the same length. In order to notice this property, students had to look at the relationship between the two sides.

On the end-of-semester survey, one student made the following recommendation in response to the question of what I could do differently in the second semester to help them be more successful in Geometry: "No computer lab! It made no sense!" This comment carries some weight in light of van Hiele levels. Based on the van Hiele pre- and post-tests, he started the semester at a level 0 and ended the semester at a level 2. Regardless of his level at the time of the activity, he was one of the better students during the semester in terms of his ability to reason about problems and to make sense of them. The fact that this task made no sense to him is significant because only 25% of the students ended the semester at a level 2. His comment provides some insight into how some of the best students (in terms of their van Hiele level) viewed the Quadrilateral Activity. Even though it was obvious to me what the students should be looking at, they were confused. They did not understand what they needed to attend to in order to identify the property.

My efficacy suffered during this task. It became obvious during the class that I had selected a task that was beyond many of the students' abilities. Initially, I was confused by their frustration. I thought that I had picked a good task and had done a good job of explaining it. This was not the case; either I had not selected a good task for my students or I had not prepared them adequately to be successful. Smith (1996) indicated that one of the moorings of efficacy in a standards-based approach was choosing problems. Perhaps the idea was good but the practices needed to solve the task were not clear to the students. I felt like I had failed my students on this task. They were not able to complete the task without a lot of assistance so it felt like a waste of time.

Appropriate van Hiele Level

I knew that I wanted to start the year with tasks at a level 0 and then move to tasks at a level 1. I had this in mind as I planned tasks but I still missed the target at times. On some tasks, I quickly realized that the level for the entire activity was too high. On other tasks, I started at the right level and was making good progress with the class but then inadvertently jumped to a higher level during the activity. The Perpendicular Bisector Activity is an example of a task that started at the right level but made a jump.

The Perpendicular Bisector Activity is an example of a task that I planned at a level 1. To introduce this task, I told my students to picture themselves in 50 years when they would be grandparents. They were about to retire and wanted to move to a location that would be as close as possible to all of their grandchildren. Assuming that one family lived in Las Vegas and the other lived in Spanish Fork, I asked my students to identify locations that would be the same distance to both cities.

Some students were confused by the question, so I provided a little help to get them started. I drew a line segment between Las Vegas and Spanish Fork on the board and asked them to identify the location where they could live in order to be the same distance from each city. They quickly identified the correct location and recognized that they knew the name for this point—the midpoint of the segment. I told them that this was just one possible location and then instructed them to try and find other locations.

Some students quickly found other points that worked and noticed that they formed a line. In fact, after I posed some questions to them, they recognized that the line was actually a perpendicular line that passed through the midpoint of the segment I had drawn. Not every student found the solution. However, after everyone had a chance to think about the problem and try their ideas, I asked a couple of students to explain their solutions.

The student explanations seemed to make sense to those who had struggled and I felt confident that we were ready to move to a new task. Before moving on, we took a couple of minutes to discuss the mathematical name for this line—the perpendicular bisector. After discussing the reasons for this name and some of its properties, we moved to the second part of the task.

The second part of the task overwhelmed them. I anticipated that they would be able to easily move from two cities to three cities, but I probably did not think very carefully about van Hiele levels. I changed the problem to include three families of grandkids—one in Las Vegas, one in Spanish Fork and one in Sacramento. When I asked them to find possible locations that were the same distance to all three of these cities, they struggled. Not one student was able to apply the idea of the perpendicular bisector to this new situation. As I look back, I think it was difficult because this required a higher level of thinking. They now had to use multiple perpendicular bisectors and make sense of the relationship between them. This was no longer a level 1 task.

These two challenges were difficult to deal with. As mentioned in Chapter 2, NCTM (1991) recommended that I think about the content, the students and the ways in which students learn in order to design tasks, but this was more difficult than I had anticipated. As illustrated above, it was difficult to plan appropriate tasks for the students that also focused on the big mathematical ideas. I tried to consider students' background and interests when designing tasks, but I was not always successful. Following the recommendations of NCTM (1991) will take lots of practice and experience.

Teacher's Role in Discourse

A couple of my new roles in the classroom also presented challenges for me. First, it was a challenge to manage the student participation. Part of this is a result of my own expectations which will be discussed later. Second, it was a challenge to be positive and upbeat during the implementation of the tasks. The significance of this challenge will be explained.

Manage the Student Participation

I began the year with the assumption that all of my students would engage in mathematics simply because a standards-based approach was more accessible for them.

A famous line from the movie *Field of Dreams* captures the beliefs that I had at the beginning of the year: "If you build it, they will come." I believed that if I built a classroom that aligned with the NCTM standards and if I gave students opportunities to make sense of mathematics, they would naturally come to a better understanding of mathematics and would be more motivated to learn it. All I needed to do was manage the participation so that everyone had a chance to participate and make comments. However, I soon realized that this ideal view was far from reality.

Due to my assumption that students would naturally do better in this new environment, I felt like a terrible teacher when some did not. I figured that it was my fault and that I wasn't doing a good job of implementing this new standards-based approach to teaching. Perhaps I wasn't managing their participation well enough. Or maybe they didn't feel comfortable enough in my classroom to participate. Perhaps they didn't know me well enough yet. In all of my attempts to understand their lack of participation, it did not cross my mind that perhaps some of my students were not interested in learning. At the beginning of the year, my state of mind insisted this was not possible.

As I met with Dr. Peterson, he assured me that I was doing a good job. He helped me to realize that I could not possibly expect every student to participate. With any approach, some students would still refuse to engage and participate. For some reason, I really struggled to accept this. In all of my reading about a standards-based approach, I had come to believe that EVERY student would be more successful. I doubt that this claim was ever made but I did walk away from the literature believing that it was a superior way to teach. And if it were a superior way to teach, I believed that it should engage all students.

I slowly returned to reality. I realized that students had their agency and could decide if they were interested in learning or not. A standards-based approach to teaching would always give them a better opportunity to make sense of mathematics and engage (NCTM, 2000), but I could not force any one of them to do so. Therefore, managing student participation became more than simply making sure everyone had a chance to participate. I had to also minimize the disruptions made by those students that did not want to learn. In a standards-based classroom, this was a challenge.

In a traditional classroom, managing disruptive behavior is solved by simply making sure that everyone is quiet and attentive. But in a standards-based classroom, groups and discussions and sense-making are all important elements. Therefore, keeping everyone quiet is not desirable. Students need the opportunity to reason together and share ideas (NCTM, 2000). Students who would have been disruptive in a traditional classroom now had many more opportunities to be disruptive in my classroom.

I noticed this behavior as I watched the videos of the Triangle Activity and the Quadrilateral Activity. Some students were simply not interested in learning mathematics. When I gave them the tasks and asked them to work to solve them, some of the students would glance at the task and then start talking about other things. For example, one student continually turned around during the Triangle Activity to talk to the student behind him. They would laugh together and make comments about the camera that was filming them. They were not focused on the task. During the Quadrilateral Activity, a pair of girls carried on a conversation during most of the class while I was walking around and helping students. I didn't notice during the actual class, but since they were right next to the camera, I caught their entire conversation. They were certainly not on the task at hand.

As I continued to journey from my theoretical world to the reality of my classroom, I somehow believed that over time my uninterested students would participate better if I kept giving tasks to them to work on and kept encouraging them to engage. I still believed that it was my fault they were not participating appropriately. As I got closer to reality, I recognized that they were not going to behave properly without some assistance. I had to help them manage their behavior. I did not have a good solution in this new setting so I moved their desks into rows and stopped doing group work. I decided that we would work on tasks as a class from that point on and discuss them as a class. In this way, I could ensure that disruptions were not taking place.

Positive and Upbeat

Another challenge for me in a standards-based classroom was being positive and upbeat. In order to better understand the significance of this challenge, first I need to describe my experiences in a traditional classroom.

In a traditional classroom, the teacher makes all of the decisions about the content, the lesson and the solution methods. The students simply listen, take notes, memorize the ideas of the lesson, practice the solution methods, and regurgitate everything on the quizzes and tests. It is easy for the teacher to be positive and upbeat because he is the only person really participating.

As a traditional teacher, I viewed the classroom as a stage and I was the performer. In order to captivate the audience of my students, I had to be positive and upbeat. I had to show them how much fun mathematics could be and include interesting tidbits of information along the way. I had to try to include games to help them as they learned the mathematics. If the students lost interest in the subject, I felt like it was my fault for not entertaining them enough. I had had the stage for the lesson and perhaps I had not performed well enough.

This approach to teaching in the traditional classroom served me well. As I mentioned in Chapter 3, I felt very successful during my first couple of years as a teacher. My students were learning the procedures and feeling confident in their abilities. They would tell me that they enjoyed the class and really enjoyed mathematics for the first time in their lives. They needed the extra energy to be successful in mathematics.

In order to be successful as a teacher in general, I realized that I would have to be positive and upbeat about the subject every day. In a standards-based classroom, I struggled to do this. A major reason for this struggle is the fact that the students were now co-performers. It was no longer a one-man show. Their ideas, their attitudes, their beliefs, their personalities and their abilities were now part of the classroom stage. I could continue to be positive and upbeat, but I was only one performer. If the majority of the performers were negative and uninterested, the show would not be very enjoyable. Unfortunately, this was the type of show that existed in my classroom on many days.

As I mentioned in the previous section on Managing the Student Participation, I found that some students were simply not interested in learning. No matter how engaging the task or how positive and encouraging I was, they were not interested in learning mathematics. These students were not about to let the rest of us enjoy mathematics. Even though they were the minority, they were vocal and affected the entire classroom with their comments. They complained about the tasks and begged for me to help them to determine a solution method. One student in class complained on a daily basis that she didn't understand anything because I wasn't explaining the concepts to her. She did not like the tasks and wanted me to just lecture and tell her everything she needed to know without making her think. She also complained that I never gave her time to work in groups. I was very confused by this comment because we spent lots of time working on tasks in groups. Later, I realized that she was referring to the typical structure of a traditional classroom where the students have time at the end of the class to start on their assignment in groups. She was not afraid to express her opinions and nearly every single comment she made was negative.

The students also whined about the homework and claimed that they never learned anything in class. One student remarked many times in class that she felt she wasn't learning very much in the class. She also described her feelings in writing on her survey when she said, "I don't feel like I learn a lot in this class." She thought the student explanations were a waste of time and continually asked me to explain the ideas more clearly. She wanted me to give her lots of facts so she could memorize them and feel like she was learning a lot.

Last but not least, they questioned why they had to learn Geometry and why we had to do tasks. One student who rarely did any work loved to ask, "So, Mr. Henry, when will I ever need to use this?" He didn't seem to have any intentions of really knowing the answer. He would always ask his questions with a mischievous grin on his face. It seemed like he just wanted a reaction from me and other students.

With this constant bombardment of negativity, I struggled to stay positive and upbeat. I relied on the students for thinking during the lesson and relied on them to bring some energy to the classroom. This rarely happened and I began to feel drained.

Another important aspect of this negativity stemmed from the mathematical abilities of the students. I found that a standards-based approach revealed a lot of student gaps in understanding. These were not gaps that were created or caused by my teaching, but gaps from previous years of mathematics. For example, most of my students struggled to solve a problem that involved a one-step algebraic equation. When I first ran into gaps such as these, I felt overwhelmed and felt that it was my fault. How was I going to teach the Geometry ideas when previous knowledge from basic arithmetic, basic Geometry and basic algebra was so weak? Should I spend my time filling in these gaps or just move on to other material? These were very real questions that I asked myself as I tried to deal with this new frustration. In a traditional classroom, I never knew what the real gaps were in my students' understanding. I just taught a new section each day and expected the students to learn/memorize it.

The negativity caused by the complaints and the gaps in understanding affected me and caused me to be more serious and negative towards the students. Instead of my usual positive and upbeat demeanor (which many students have commented on in the past), I was serious, boring and negative. This is evident in the videos of the Triangle Activity and the Quadrilateral Activity. I did not look like I was enjoying myself and I rarely smiled. At one point during the demonstration of the Quadrilateral Activity, I asked the class a question about the quadrilateral that was projected on the wall, and I didn't get any response. I could see in my face the frustration I felt. I was doing everything I could to involve them and to get them thinking and I was not very successful. I thought for sure that the technology would interest them, but this was not the case.

The classes in general were really boring to watch. It was obvious to me as I watched myself teach that I was not comfortable in my new roles and that I struggled to feel like I was teaching effectively. When I feel that I am doing a good job, I am happy and confident.

In order to regain some of my energy in the classroom, I moved the students back into rows and directed the tasks as a class. If I could minimize their part in the show, I could maintain a positive, upbeat demeanor and classroom. Even if they continued to have negative thoughts, those thoughts would not affect me and I could help make mathematics enjoyable for those who also enjoyed it. I could also help those who struggled with mathematics but who were willing to try and learn. I didn't have a good solution to help students fill in their gaps but at least there was more of a positive energy in the classroom.

I felt like a much better teacher when the overall environment was positive and upbeat. I felt like students were more successful. I felt like students spent more time working rather than complaining. And most importantly, I was happy at the end of the day and excited to plan lessons for the next day. Without the positive energy, planning for the next day was drudgery and something I did not look forward to.

Knowing about my new roles (Lester et al., 1994) as described in Chapter 2 helped a little bit, but I also recognized the difficulty of this transition. Even though I focused on eliciting student reasoning, listening to student thoughts and managing student participating, it was still harder than I anticipated. These new roles take lots of practice. They are very different from the traditional roles and require time to develop.

Students' Role in Discourse

The main challenge of this standard from my perspective was effectively teaching students about their new roles. As I was trying to better understand my own roles, I had to also help students understand what was expected of them. It was a challenge to teach them the importance of explaining their reasoning and the importance of listening to, questioning and learning from other students.

Explaining their Reasoning

The first challenge of teaching students about their new roles was helping them understand the value of explaining their reasoning. This role was important to help them to build towards proof. In order to be able to explain their reasoning, they first needed to be able to learn how to reason on their own. This was a foreign skill for them in mathematics. In a traditional classroom, students were often expected to explain the steps to a procedure which was essentially regurgitating facts that the teacher had presented earlier. Rarely were they given the opportunity to work on an open-ended task, reason about possible solutions and then explain to others how they reasoned through the problem. Because this was such a new concept for them, they understandably struggled with it and felt frustration because of it.

In order for students to be able to explain their reasoning, they first had to learn how to reason about problems themselves. In one of my journals, I wrote about a conversation that I had with a student after school concerning her reasoning. She had just finished making up a test and I asked her how it went. She expressed frustration because she felt the test was too hard. She indicated that if I would just tell her what to do on each problem, she would do better. This illustrated the fact that many students were not comfortable thinking or reasoning for themselves and wanted me to spoon-feed them the information. They were used to being spoon-fed in traditional mathematics classes and had to adjust to this new approach which required some thought.

In another journal entry, I expressed the hope that more students would buy into this thinking as the year went on. I wasn't quite sure how to effectively teach them about this new role but I continued to ask why questions and listen to student thinking. Some of their comments on the survey indicated to me that they still did not value the explanations of reasoning. One student described our classroom by writing, "By having students explain and becoming disinterested in the subject by taking an hour to explain, when you could've explained it to us better." Another student wrote, "Maybe explain and not wait so long for us to figure it out, because then we waste time and you lose our attention towards the middle of the discussion." These comments indicate that a discussion of reasoning was boring to them and a waste of time. They just wanted me to explain the problems and do it more quickly. Student reasoning took too long for their liking.

One of the strategies that I used to promote thinking and reasoning was to continually ask students why they decided on a certain answer. I would ask questions such as, “Why did you do this?” or “Can you explain why that made sense to you?” After a student explained their solution to a problem, I would typically say something like, “That could be it. What do you think class?” I would ask the class to decide on the correctness of the explanation rather than assessing it myself. One student disagreed with my questioning and recommended that “instead of saying, ‘That could be it. Why?’, say what it is, not what it’s not. We talk more about the wrong way of doing things than the right way. It is so confusing.” This student apparently wanted me to decide if a student’s approach was correct before spending time talking about it as a class. She was not comfortable reasoning about the ideas herself.

The discomfort produced by my constant efforts to encourage thinking and reasoning was evident during class discussions. This period of time seemed to be the most difficult for my students. They seemed to enjoy the activities but did not seem too excited to discuss the big mathematical ideas afterwards. The discussions were hard for them because I wanted them to explain their reasoning from the activities.

I wrote in my journal about an example of this. In that entry I described my thoughts about a demonstration in class. We were discussing similar triangles and I guided students through a demonstration of how we could measure the height of the door using a mirror placed on the ground. One of the students looked into the mirror until he could see the top of the door and then we measured distances and heights. I thought this was an interesting idea that could be applied to measuring large objects such as a tree outside.

After the demonstration, I drew a diagram on the board and asked questions to help them make sense of the mathematics behind the demonstration. I listened to their ideas and helped them make connections. During this conversation, I noticed many looks of boredom on their faces. They enjoyed the demonstration but struggled to talk about the mathematics behind it. The discussion was not as comfortable for them, possibly because they struggled to explain their reasoning.

I noticed similar results when watching the video of the Triangle Activity. The vast majority of the students were actively engaged in the activity. They were trying to construct different types of triangles and asking friends for help when they got stuck. They were eager to show me their triangles and wanted to be involved when presenting. However, once we finished constructing the triangles and cutting them out, their level of interest dropped. They were not skilled in reasoning about the results that they had found. The important mathematical findings of the activity were hard to talk about.

In summary, I was not comfortable teaching students how to explain their reasoning. They had not had opportunities in the past to reason through open-ended problems and to explain their thoughts to others. As a result, class discussions were uncomfortable for me and for many students. They simply were not prepared to engage in a conversation about their own thinking and reasoning.

Listening to, Questioning and Learning from Other Students

This student role of listening to, questioning and learning from other students is closely related to the previous role of explaining their reasoning. Students need to learn to explain their own reasoning and they also need to learn how to listen to other students' reasoning, question it and learn from it. In a traditional classroom, this rarely happens. Students explain the steps of a procedure and other students listen, but this is merely a review of what the teacher already taught. Rarely do students introduce new ideas with their explanations. This occurs in a standards-based classroom and puts pressure on the other students to listen and question and try to learn from these student explanations.

It was a challenge for me to know how long to continue a discussion of these student explanations. For some students, a five-minute discussion would be sufficient. For others, a discussion for an hour would still not be long enough to understand the reasoning. These thoughts showed up on the survey. One student wrote, "A few things we spend too much time on. I would like to move a little faster. I feel like we're behind the other Geometry classes." Another student recommended that we should "explain things more, make sure everyone understands, don't zoom through everything." Regardless of the length of the discussion, some students would not be happy listening to and learning from other students. Some would be bored if we took an hour and others would be overwhelmed if we took five minutes.

It was certainly a challenge to teach students to question and learn from other students. They had been trained through the years to look to the teacher to validate their thinking but were now expected to be part of the validation process. Their frustration with this new role came up as a comment on the survey. One student wrote, "Make the solutions more clear because sometimes when it comes out of the student's mouth it's not very clear to me." They were not used to hearing solutions for the first time from another student and struggled to understand them. Rather than ask the student questions to better understand the method, they wanted me to translate for them.

Ultimately, the students wanted any new ideas to come from my mouth rather than a student's mouth. In describing the approach to learning in our classroom, one student wrote, "Everybody talks together about it which I never learn anything because I can't hear the teacher. Try to get everybody to be more quiet so I can hear you and understand what's going on." Even though students were explaining ideas all around her, she didn't want to listen to them. She only wanted to listen to what I had to say. However, I tried not to say too much in order to let students explain their own reasoning. She did not enjoy this setting and preferred the traditional approach to learning.

I wanted students to learn how to listen to each other and learn from each other so I often redirected student questions back to the class. On many occasions, another student was able to answer the question. If another student was not able to answer the question, I would ask the student who asked the question what he thought the answer might be. I was hoping to get them to at least make an educated guess and put some thought into the possible answer. Amazing as it was, on many occasions they answered their own question. They didn't always like that, but it showed me and them that they could answer a lot of their questions by putting forth a little effort to think and reason about a possible answer. One student expressed his dislike for the questioning by writing on the survey, "Don't tell us to answer my own question so much." It wasn't comfortable for them to think and reason about their own questions.

I struggled to help students learn and understand their roles to listen to, question and learn from others. As discussed in the previous section, this is a difficult skill for them to learn. It is new to them because they haven't had to think for themselves in a mathematics class before. This newness leads them to be uncomfortable as they try to reason and listen to others' reasoning.

These new student roles (Hiebert et al., 1997) as described in Chapter 2 were just as hard for me as they were for the students. It was a challenge to know how to help them develop the roles. I could certainly read them my expectations, which would be based on the literature, but I did not have enough experience to know how to help them develop the roles.

Chapter 5 – Discussion

An important part of action research is developing a plan of action. The plan of action helps me focus on what I can do next year to overcome some of the challenges that I faced this year. The plan of action also provides other teachers with information that may be helpful to them as they attempt to transition from a traditional approach to more of a standards-based approach.

In order to develop the ideas for my plan of action, I want to focus on just two significant challenges that I experienced during the semester. Then I want to identify some of the things that I did to try and overcome these challenges. After discussing what I did, I want to discuss some successes from the semester that need to be taken into account before describing my plan of action.

Two Significant Challenges

There were many challenges that I experienced during the semester that were expected but some were not. I expected to struggle in developing tasks at the right van Hiele level. I expected that both the students and I would struggle in our new roles but I wasn't sure what that would entail. Both of the significant challenges that I will discuss provide insight into the challenges that are involved with learning new roles.

These two significant challenges were related to each other and both came as a result of my emphasis on involving students in the classroom and listening to their reasoning. In a standards-based classroom, we want to give students ownership in the learning process and help them to make sense of the mathematics. However, this student involvement can adversely influence the learning environment.

First, in my experience, student involvement brings with it whining, complaining and negativity. As I discussed in Chapter 4, I view the classroom as a stage. In a traditional classroom, the teacher is the sole performer and thus has a great influence on the learning environment. The atmosphere and the energy in the classroom are ultimately determined by him. However, when we invite students to join us on the classroom stage, we give up a lot of our control over those elements. The students now contribute their feelings and opinions regarding mathematics. In the case of my particular students, these feelings were usually negative. It is difficult to maintain a positive learning environment when negative comments are shared on a regular basis.

Second, student involvement reveals gaps in understanding. In a standards-based approach, it is expected that students will share their thinking. As we give students the opportunities to contribute their ideas and their thinking, we quickly recognize when they are missing important ideas. This happened on many occasions in my classroom. I assumed that they understood certain ideas like how to solve a one-step algebraic equation. However, as we discussed this idea and others, it became apparent that many students had no idea what to do.

This finding is significant finding it is very different from the traditional approach. In the traditional approach, students are expected to quietly listen to the teacher. They are not given the opportunity to share their thinking. They take notes on the teacher's thoughts and then regurgitate this same information on homework, quizzes and tests. Because of the expectation for students to quietly listen, it appears that the traditional approach actually hides students' lack of understanding.

Since the traditional approach hides students' lack of understanding and a standards-based approach reveals students' lack of understanding, teachers-in-transition will struggle in this area with their efficacy. Similar to my experience, teachers-in-transition who see all of these gaps in understanding will at first think it is due to the approach. They will think it is their fault and will struggle to recognize that this approach is simply revealing gaps in understanding that were already there.

These two challenges are significant because they were unexpected and they shed light on a very important component of a standards-based classroom. Involving the students more in the classroom has side effects that have to be considered and dealt with. I will discuss what I did during the semester to try and overcome these challenges and also what I plan to do next year.

What Did I Do?

As mentioned in Chapter 4, I tried to deal with these challenges by putting students in rows and limiting the amount of group work. I felt like the classroom was more positive because I was able to manage the disruptive students better. They didn't have as many opportunities to talk. By limiting the students that were allowed to join me on the classroom stage, we had more energy and the class was more upbeat.

However, putting students back in rows did not help to fill in the gaps that they had in their understanding. In fact, I don't think I really recognized what was happening until late in the semester. At the beginning of the semester, I just assumed that I was doing a poor job as a teacher every time a gap showed up during class. I really felt like it was my fault that they were struggling so much. After a month or two, I started recognizing that their gaps were due to previous mathematics classes and could not possibly be my fault. Over time, I realized that I was doing a pretty good job of following the approach but I just didn't recognize that one of the side effects is the revealing of student gaps.

I focused on trying to maintain an environment that would help me maintain a positive teacher efficacy. However, more recent research reveals that positive teacher efficacy does not always improve the teaching in a classroom (Wheatley, 2000). Wheatley (2002) identifies that a combination of positive teacher efficacy and teacher efficacy doubts might be best for teachers and the reform in general. Teachers might put forth more effort to learn and improve their teaching when they have doubts about their efficacy. Wheatley (2002) also points out that teachers need to stay motivated to learn because there is so much to learn about a standards-based approach. If they become too comfortable in their teaching, they may not try to continually improve their teaching. I certainly recognize that teacher efficacy doubts motivated me to think more deeply about my teaching. However, I would be concerned that teachers would have a hard time teaching day after day without a strong sense of positive teacher efficacy. I submit that an appropriate balance would involve mostly positive teacher efficacy with an occasional teacher efficacy doubt to keep the teachers thinking and improving.

Successes

I am now stepping outside of the typical thesis format to address a certain population. Chapter 5 is not usually the place to describe new data, but I will make an exception in order to give teachers a better perspective of my experience. There were many challenges but there were also many successes. I have only focused on challenges up to this point because it was the challenges that answered my research question. However, since successes are also important in determining a plan of action, I will spend some time with them here. The successes involved mathematical understanding, student roles and student perceptions.

Mathematical Understanding

I want to identify two successes here. First, students at times reasoned through a formal proof by simply explaining their reasoning. Second, students did just as well on the end-of-level state test as students in a traditional classroom.

First, students showed me at times that they could produce a formal proof just by explaining their reasoning. During one class, we were discussing vertical angles. We discussed how to identify them and also why they were called vertical angles. On a whim, I decided to ask my students if they could explain why the vertical angles were congruent. One of my students raised her hand and explained that both of the angles were supplementary to the same angle and therefore had to be congruent. We wrote down each of her ideas and essentially had a formal proof on the board. I was very impressed with her ability to prove a mathematical idea by simply explaining her reasoning.

Second, my students did just as well on the end-of-level state test as students in the traditional classes. There were two other Geometry teachers in the school and both taught according to the traditional approach. When we received our end-of-level test scores, I was anxious to see how my students had done in comparison. The raw average of all of my students was 63.67%. The other teachers had raw averages of 55.93% and 63.94%. I was thrilled to see that my students were able to do just as well as or better than the other students. Because it takes longer to learn material in a standards-based approach, we did not even cover all of the material that the other teachers covered, yet my students understood the material that we had covered more deeply.

Student Roles

There are two examples of successes in student roles that I want to share. First, students became more and more comfortable explaining their reasoning over the course of the semester. Second, there was an obvious difference between students who transferred into my classes during the semester and students who were in my classes from the beginning of the year.

First, students became more comfortable explaining their reasoning over the course of the semester. In my journal, I discuss an experience in class when I posed a question. One of my students raised his hand and gave a quick answer but then said "because". Before he could continue, I cut him off to point out to the class how proud I was of him. Instead of just giving his answer, he was ready to explain his reasoning and defend his answer. As it turns out, by the time I let him continue with his thought, he had forgotten what he was going to say. Over the course of the semester, students explained their reasoning more and more without having to be asked. They were gaining a better understanding of their role.

Second, there was a difference between students who transferred into my classes and those that started the year in my classes. I remember on one occasion asking a question and getting the response, “because you told us.” The student who responded had just transferred from a traditional class and did not understand the student roles or my expectations. I briefly explained to this student that we were trying to make sense of the mathematics by explaining our reasoning. The other students then provided good examples of trying to reason through the question. There was a big difference between the two types of students. This showed that the students were becoming more comfortable in their roles and understood the importance of explaining their reasoning.

Student Perceptions

As the semester came to an end, I asked each student to fill out the survey in order to provide me with some student feedback. I was honestly contemplating going back to the traditional approach if the students indicated that they wanted it on the surveys. It had been such a rocky semester and I figured it was because of the approach. I was willing to do whatever the students wanted me to do just so I could feel more successful. I expected lots of complaints and lots of requests to go back to the traditional approach. But the feedback was very different from what I expected.

On the survey, I asked the students to rate their ability to learn through lectures (traditional approach) and through new problems and student solutions (standards-based approach). I was expecting more students to say that they learned better with the traditional approach because there had been so much criticism by the vocal minority about a standards-based approach. However, 48% of the students rated a standards-based approach higher than the traditional approach and 37% rated them equally. Only 15% of the students actually rated the traditional approach higher than a standards-based approach. This information was very comforting. Even though some students complained, the majority of the students were pleased with the new approach. The research says it is a superior way to teach and the students prefer it even though I wasn't doing it perfectly. They were still able to recognize how helpful it was to their learning and understanding.

This information alone made me feel like the semester had gone much better than I had thought. I was beating myself up for my shortcomings and my inexperience in my roles and yet the students preferred what I was doing. Some of the general comments on the survey included: "we discuss a lot. I think that helps"; "we have made it make sense by thinking through the problems."; "we've been learning by using real-life problems, then solving them as a group. It's a lot easier to learn this way than doing problems from a book."

Other students were turned on to mathematics because they finally had the opportunity to think for themselves rather than being told what to do. One student told me on a couple of occasions in class that she really appreciated the discussions because they helped her to make sense of the mathematics. Her mother emailed me during the semester to tell me that mathematics was making sense to her daughter for the first time in her life. She also mentioned that her daughter was working with a tutor to help her master the material. The tutor really liked the approach I was using to teach Geometry.

She commented that if the daughter stuck with it, she would have a much deeper understanding of mathematics than she could gain in the traditional classroom. It was neat to hear this as a result of using a standards-based approach.

Plan of Action

Given the challenges as well as the successes just mentioned, I now want to discuss my plan of action for next year. First of all, I will be more confident next year because the students have made it clear that they prefer this method of teaching. We will still have issues to work through, but in the end they will be pleased with the learning experience. Second, I now know that I cannot expect every student to succeed. This will make the beginning of the year much better for me. I will not be so stressed wondering why certain students are not learning. I will give them the opportunity to make sense of mathematics but will still respect their agency. Confidence and realistic expectations will provide a nice framework for me to deal with the two significant challenges, which are managing students' negativity and dealing with gaps in understanding.

Student Negativity

My plan for managing students' negativity involves starting the school year in rows. I will have to slowly introduce them to the classroom stage and make sure they know their part before letting them perform. I plan to give individual tasks at first and have students write down some of their thoughts. I will look through the comments and read some that are good examples of what I expect. We will use these examples to talk about how to explain reasoning. As the majority of the students master this exercise, we will start sharing thoughts in partnerships.

There will be two main goals for partnerships. First, I want them to practice explaining their reasoning verbally. Second, I want them to practice listening to, questioning and learning from their partner. To encourage all of these goals, I plan to provide them with an evaluation form. I will ask each student to evaluate themselves and their partner in terms of how well they explained their reasoning. Then I will ask them to explain what they learned from their partner as they asked questions. We will continue doing these types of activities until the majority of students feel comfortable explaining their reasoning and also learning from others' explanations.

Once the students have proved that they understand their new roles, I will begin to allow them to contribute to classroom discussions. I believe that this will help the classroom to have more of a positive and upbeat atmosphere. They will be trained in their new roles and will know how to succeed as a student. When the students have opportunities to share their thoughts, they will know what is expected. Also, the other students will know that they are expected to listen, question and learn. I anticipate that the number of negative comments will decrease because of this approach.

Gaps in Understanding

Dealing with gaps in understanding is going to be an ongoing challenge for me. My plan involves developing tasks that start at a very basic, elementary level. This will give me the opportunity to deal with gaps in understanding before we get to the Geometry content. As I implement tasks, I will get a better idea of the specific building blocks that are needed and modify the tasks accordingly for future years. I think that this challenge will take years to really overcome but I can certainly make a lot of progress next year. At least now I know that gaps will appear and I will be looking for them.

Recommendations

There are a few recommendations that I would make to other teachers-in-transition. First, recognize that this approach is not a guaranteed approach to help EVERY student; rather, it gives every student the opportunity to make sense of mathematics (Hiebert et al., 1997). Some will choose not to take the opportunity because it is new to them and hard. Others will choose not to take the opportunity because they're not interested in learning. Students still have their agency and you can't force all of them to learn even if you are using a better approach than before. However, if done well, many students will be turned on to mathematics for the first time and students overall will gain a deeper understanding.

Second, recognize that this approach takes a lot more planning time. Developing tasks and carefully considering student thinking and background requires a lot of time. Developing homework assignments that build on the in-class tasks also requires time. I would recommend taking an entire summer to prepare for this type of change. Spend time reading research and discuss your ideas with other teachers. Then identify and develop tasks. Finally, prepare yourself mentally for a much different experience.

Third, be flexible and recognize that you and the students are taking on very different roles (Hiebert et al., 1997; Lester et al., 1994). If you need to give some hints to keep students going, do it. If you need to stop a task and get rid of it completely, do it.

You are not going to implement this approach perfectly. In fact, it will probably take several years before you have developed the tasks just how you want them and have developed the learning environment in just the right way. However, you will be convinced after just a couple of good tasks that students learn so much more and understand the mathematics so much better.

Limitations of the Study

In any research study there will be limitations of some kind. In my study, one thing that may have hindered the findings was my limited experience teaching Geometry. Even though I had taught for a few years previously, this was just the second year I had ever taught Geometry. My first experience teaching Geometry occurred four years previously. In that school, there were two levels of Geometry classes—honors classes and regular classes. Proof was taught in the honors classes but not the regular classes. I taught three regular classes and therefore did not have a lot of experience in teaching students to think about proof. As I prepared to teach Geometry this year, my insights into student thinking came only from research and not actual experience in the classroom. A little more experience with the concepts of Geometry may have helped as I developed tasks and led discussions.

Applicability of Findings

Some of the findings for this study are applicable to any teacher-in-transition. However, other findings would only be applicable in classes where the students are average or below average.

All teachers-in-transition will experience discomfort as they adjust to new roles and measure efficacy in different ways. It would be helpful to recognize some of the hurdles in advance and identify ways to jump over them. It is also helpful to recognize some of the side effects that come with the new approach.

In classes where students are average or below average, teachers can expect that their students will not be confident in their mathematical abilities and will therefore need a lot of support as they try to learn new roles. A lot of patience will be required because they will not naturally jump into the tasks and engage themselves. They need a lot of reminders and encouragement and they need positive reinforcement.

Ideas for Future Research

One idea for future research would be to do this same action research project again, using the knowledge that I have now. I would have a better idea of what students would struggle with and would be able to create tasks that would help them at their level. I would also do a better job of teaching students about their roles and being confident in my own.

Another idea would be to repeat this action research project in a different class such as Algebra 1 or 2. Proof is not such an emphasis in these classes, but the ideas of reasoning and justification are still important. In fact, since 10th graders taking Geometry are not really prepared to talk about proof anyway, the focus on reasoning and justification would be about the same in any of the classes.

One last idea would be to repeat this same study, but in an 8th grade or 9th grade Geometry class. It would be interesting to compare the results between the different levels and look to see if students that take Geometry sooner are better able to reason and justify and can actually arrive at a van Hiele level 3 by the end of the school year.

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Appendix A

Geometry Lesson Plan Outline

Lesson/Objective	Activity/Approach
1. Recognize the need for undefined terms	a. Use exercise involving definitions in a dictionary (discuss circular definitions). b. Discuss undefined terms—words that are undefined, so that we can build other definitions on them.
2. Organize shapes; think about properties of triangles (VHL 0-1)	a. Burger's exercise on organizing triangles b. Define triangles; ask students to define terms they use in the definition of triangles; reach a point where we are forced to identify some undefined terms. If we end up defining point, line and plane as undefined terms, we will have covered section 1.1 of the book. c. After dividing up the triangles, ask the groups to provide a description of each grouping. Exchange these descriptions with other groups and see if they divide up the triangles in the same way
3. *Identify Van Hiele levels of thinking for each Geometry student*	a. Take Van Hiele test during the 2nd week of school.
4. Explore and classify types of angles (VHL 0-1)	a. Use parts of Miller's activities on "Lines, rays, and angles" (pp. 4-7) and "Measuring angles" (pp. 8-11); this is also covered in section 1.4 of the book. b. Talk about what 1 degree is. What if we defined the measurement around the circle to be 12 Gregs? Talk about what 1 radian is. Use sticky string stuff to go around concentric circles. Notice that the angle for 1 radian is the same for both circles, etc.
5. Explore the angles in a triangle and classify the triangles based on the types of angles they have (VHL 1)	a. Use parts of Miller's activities on "Triangles" (pp. 20-22) and "Angles of a triangle" (pp. 25-26); this is also covered in section 4.1 of the book.
6. Explore the sides of a triangle and classify the triangles based on the lengths of its sides (VHL 1)	a. Use parts of Miller's activity on "Equilateral and isosceles triangles" (pp. 27-28); this is also covered in section 4.7 of the book.
7. *Test/Activity covering undefined terms, properties of triangles, classifications*	

of angles and triangles	
8. Introduce/explore the idea of triangle congruence (VHL 1?) 9. Identify applications of triangle congruence—fence gates, bridges, triangular-prism-shaped packaging boxes.	a. I give the students some dimensions and see what they come up with. For example, give two sides and an angle, but don't instruct them to put the angle in a particular place. Ask them to make as many triangles as possible. I will also use SSA to instill some uncertainty as to whether the other types (SSS, SAS, ASA, AAS, HL, AA) are congruent all the time. Note: Use large dimensions with the triangles to decrease the percentage of error. We will touch on sections 4.3, 4.4, 4.5 and 6.4, but we will come back to this idea later and prove it. b. Notice that some are postulates and others are theorems. Distinguish between the two. Postulates (like Euclid's postulates) are statements that are believed to be true. Based on these statements, we can prove other statements, which we then call theorems. c. Discuss congruence (same shape and size) and similarity (same shape) afterwards and create definitions (sections 1.2, 4.2 and 6.3). Students might define congruence in terms of transformations (section 4.8).
10. Become familiar with the properties of quadrilaterals by using Geometer's Sketchpad to explore. 11. Become familiar with other quadrilaterals using Geometer's Sketchpad. 12. Explore some quadrilaterals, with a focus on creating some doubt as to when students are sure they know the shape (VHL 1)	a. Draw a parallelogram (or some other quadrilateral) for the students. Use the measuring tools on GSP to measure everything they can. Give a prize for the person that finds the most properties. b. Use Burger's activity of "What's My Shape". Read off some clues, each time asking if the students know which quadrilateral I am talking about. This will bring up the issue of necessary vs. sufficient information. Touches on 1.6.
13. *Test/Activity/Project involving triangle congruence and quadrilateral classifications.*	
14. Explore and classify angle pair relationships (VHL 0-1)	a. Use parts of Miller's activity on "Angles with the same vertex" (pp. 12-15); this is also covered in section 1.5 of the book. Discuss collinear.
15. Define parallel, perpendicular and skew	a. Draw a picture of two lines. Now draw a

lines. 16. Identify angles that are formed from a transversal intersecting two lines. 17. Identify that the converse is also true (later on, look for a case where the converse is not true). (VHL 0-1)	different picture with two lines. How many different kinds of pictures can you draw? Describe how each picture is different. Discuss coplanar. b. Draw two lines that are parallel. How did you do it? Now draw a line that crosses both of the parallel lines. Measure all of the angles. What do you observe? c. As a side note, talk about drawing parallel and perpendicular lines using the compass. d. Draw two lines that will intersect somewhere off the page. Now draw a line that crosses both of the parallel lines. Measure all of the angles. Can we conclude the same things that we concluded from the parallel lines? e. Also look at parts of Miller's activity on "Parallel and Perpendicular lines" (pp. 16-19); these activities cover 3.1, 3.2 & 3.3.
18. *Test/Activity/Project involving angle pair relationships and transversals.*	
19. Introduce and use the mid-segment theorem. (VHL 0-1)	a. Use the compass to bisect each segment of a triangle (find the midpoints); what do you observe about these mid-segments (joining the midpoints together)? Measure the angles and the sides and make conjectures; covers 5.1.
20. Introduce and use the perpendicular bisector. (VHL 0-1)	a. Use the compass to draw perpendicular bisectors to the three sides of a triangle; what do you observe? Define circumcenter and equidistant—all points on the perpendicular bisector are equidistant from the vertices or endpoints. This covers 5.2. b. Problem involving grandparents wanting to live the same distance from all three of their kids/grandkids. Given three locations, where should the grandparents move? c. Problem involving building a park (or an airport) in the county the same distance from the three largest cities (see p. 317 #1).
21. Introduce and use angle bisectors. (VHL 0-1)	a. Use the compass to bisect the angles of a triangle. What do you observe? Define incenter. Covers 5.3. b. Problem involving the need to place a fountain in a triangular pond so that it is

	equidistant from each of the sides of the pond (see p. 315, #29). c. Problem involving cutting the largest possible circle out of a piece of wood (see p. 317 #2). d. Good error analysis problems (see p. 314, #21-22)
22. Introduce and use medians and altitudes. (VHL 0-1)	a. Problem: Given a triangle, where should I place my pencil to balance it? See the activity on p. 318. b. Draw the medians (finding the centroid); section 5.4. Use GSP for precision. Measure everything and discover the 1/3 and 2/3 properties. c. Draw the altitudes (finding the orthocenter). Use GSP for precision.
23. Use inequalities in a triangle. (VHL 0-1)	a. Problem involving a misprint on a map (see p. 333, #39); covers section 5.5. b. Given three lengths, try to form a triangle. There is an activity involving rugs in IMP Year 2 (pp. 226, 230).
24. *Test/Activity/Project involving attributes of triangles—mid-segments, altitudes, medians, angle bisectors and perpendicular bisectors.*	
25. Discuss more about similar polygons using AA; extends from lesson 8. (VHL 1) 26. Define similarity in mathematics. (VHL 1)	a. Similarity activity (Dr. Peterson's book p. 724). Write a description of what is meant by "This object is similar to that object". Exchange cards. Based on the definition on the card in front of you, are the following similar? Discuss the conflict of English meanings of words vs. mathematical meanings of words. b. Compare a tennis court to table tennis (see p. 378, #31). Are they similar? c. Compare the computer screen in the classroom with the projected image (see p. 378, #32). Are they similar? d. Air Hockey (p. 386, #31). e. Covers 6.3 and 6.4.
27. Think about similarity using SSS or SAS Similarity (VHL 1-2).	a. I have a triangle of such dimensions. Make a triangle that is similar. How did you approach it? b. Set up a problem that would prove the mid-segment theorem using similarity.

	Using my GPS system, I want to bury four new caches. My starting position is at the intersection of two straight roads. The first two I plant along one of the roads at the 10-mile mark (cache A) and the 20-mile mark (cache B). I return to my starting point and plant two more along the other road at the 8-mile mark (cache C) and the 16-mile mark (cache D). How much farther apart are caches B & D than caches A & C? How do you know? c. What if I placed cache B at the 30-mile mark and cache D at the 24-mile mark. How much farther apart are caches B & D than caches A & C now? How do you know?
28. Discuss ratios and proportions (*these are very important ideas; they help students in many areas of mathematics, namely slope. This is fundamental and needs to be explored deeply*). (VHL 1)	a. Go outside and try to determine the height of a tree by looking at our shadow and the shadow of the tree. You can also put a mirror on the ground to measure the height. Move until you can see the top of the tree in the mirror. You can also hold a protractor that has a pencil tied to string. Find the angle. b. Billiards problem. You want to hit a ball by bouncing off one of the sides. Reflect the other ball over the side and hit towards it. You have congruent triangles and you can show that the angle of incidence is the same as the angle of reflection. These cover sections 6.1 and 6.2. c. Look at a map and determine the total distance based on the scale (see p. 366, Example 4). d. Scale model of a 3-D scale model of the Reunion Tower in Dallas, TX (see p. 366, Example 5). e. Longaberger Company Home Office that looks like a basket they sell (see p. 368, #22). f. Statue problem. Compare the statue to the real person. Compare the arm to the arm or the nose to the arm. Talk about the differences. Corresponding sides vs. corresponding ratios.
29. Explore proportionality in triangles.	a. Use activity involving GSP on p. 396.

(VHL 1-2)	b. Lake frontage problem (see p. 402 #24)
30. Use knowledge of similar shapes to create fractal drawings (self-similar relationships on segments and angles, not necessarily on the whole shape). (VHL 1).	a. See the extension on p. 406 to make a Koch snowflake. b. Ask Marie about her complex numbers activity involving fractals (pre-calc class?)
31. Perform similar transformations using dilations (*Warning—make sure this fits nicely here; otherwise, move it to Chapter 9 when we discuss transformations. Also, look at the Chicago stuff; they start similarity stuff with dilations*). (VHL 1)	a. See the Dilations activity on p. 408. b. Use the overhead to enlarge a drawing. c. Create a perspective drawing using dilations (see p. 415 #31). d. On a coordinate plane, draw a block letter. Use dilations to draw a shape twice as big. How do we know they are similar?
32. *Test/Activity covering similarity in figures, fractals, and dilations.*	
33. Revisit triangle congruence. (VHL 1-2)	a. Activity involving a job: you have to ensure that every triangular piece of glass is congruent. How would you do this? b. Can you think of a way to find the distance across a river or a canyon using triangle congruence (see p. 261 #28)? How about using triangle similarity? Which would you prefer? c. Given five triangles with different markings, which are congruent (see p. 260 #27)? d. Given a right triangle, what would you need to know to prove two triangles congruent (see top of page 252)? e. These cover section 4.6.
34. Revisit transversals and parallel lines. (VHL 1-2)	a. Set up a problem which would end up proving the triangle sum theorem. b. Tear off the corners of a paper triangle and move them to the top angle. Draw the line formed by the three triangles. What do we know about the base of the triangle and this line (they are parallel)? How do we know (alternate interior angles)?
35. Discuss parallel/perpendicular lines in terms of slope. Introduce coordinate geometry (similar to Algebra 1). (VHL 1-2)	a. Covers 3.4, 3.5, 3.6, and 5.1. b. Draw a line, find the slope of that line. Draw a line perpendicular to that line. What is the slope of the perpendicular line? How do the two slopes relate? Why do you think that works? Draw right triangles to associate them. c. See Dr. Peterson's book p. 782. Draw in

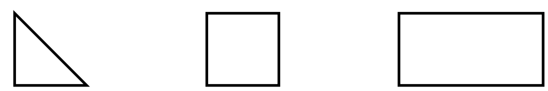
	the triangles and rotate them.
36. Explore the Pythagorean Theorem and relate this to the distance formula. Also explore the midpoint formula. (VHL 1-2)	a. Visual activity with the Pythagorean Theorem. b. Identify where the distance formula comes from.
37. Use the distance formula and midpoint formula to make conjectures in a coordinate geometry environment. (VHL 1-2)	a. Classify shapes. Verify theorems from Chapter 5.
38. *Test/Activity covering congruence, parallel lines, Pythagorean theorem, distance formula, coordinate geometry..*	
39. Identify Van Hiele level of thinking for all Geometry students after having spent time thinking about reasoning, justification, and proof	a. Take Van Hiele test again right before Christmas.

Appendix B

van Hiele Geometry Test

(CDASSG Project, University of Chicago, 1980)

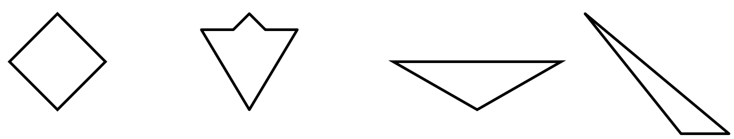
1. Which of these are squares?



K L M

A. K only
B. L only
C. M only
D. L and M only
E. All are squares

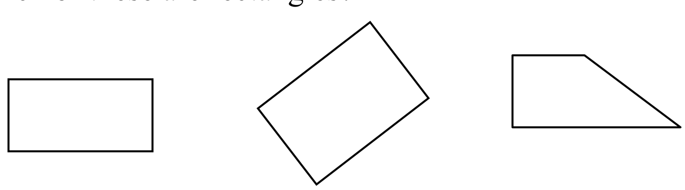
2. Which of these are triangles?



U V W X

A. None of these are triangles.
B. V only
C. W only
D. W and X only
E. V and W only

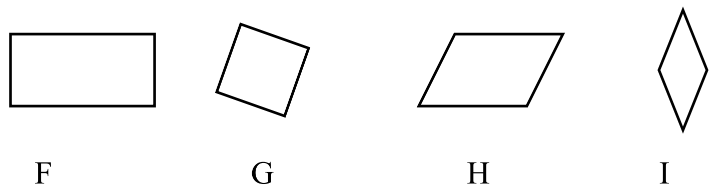
3. Which of these are rectangles?



S T U

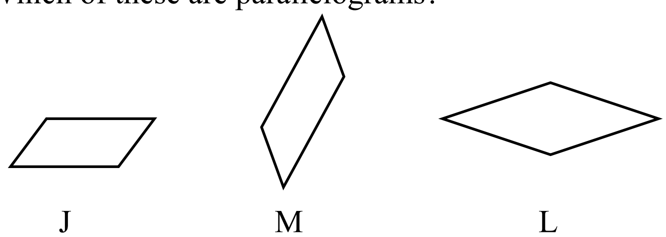
A. S only
B. T only
C. S and T only
D. S and U only
E. All are rectangles.

4. Which of these are squares?



A. None of these are squares.
B. G only
C. F and G only
D. G and I only
E. All are squares.

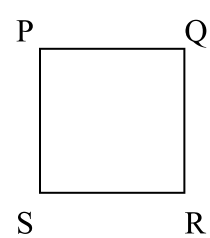
5. Which of these are parallelograms?



A. J only
B. L only
C. J and M only
D. None of these are parallelograms.
E. All are parallelograms.

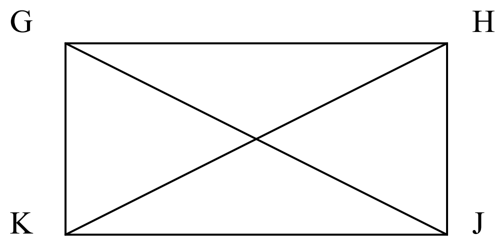
6. PQRS is a square.

Which relationship is true in all squares?



A. $\overline{PR}$ and $\overline{RS}$ have the same length.
B. $\overline{QS}$ and $\overline{PR}$ are perpendicular.
C. $\overline{PS}$ and $\overline{QR}$ are perpendicular.
D. $\overline{PS}$ and $\overline{QS}$ have the same length.
E. Angle Q is larger than angle R.

7. In the rectangle GHJK, $\overline{GJ}$ and $\overline{HK}$ are the diagonals.

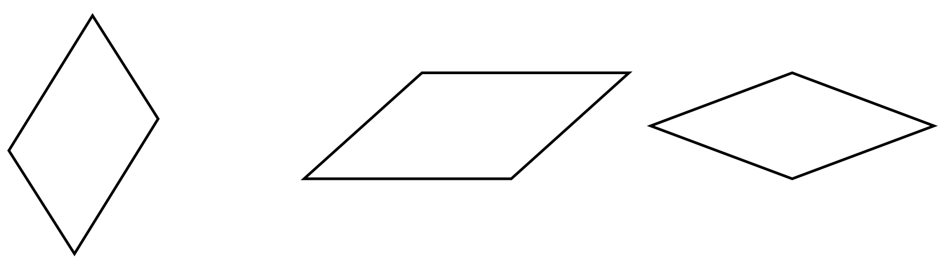


Which of (A)-(D) is not true in every rectangle?

A. There are four right angles.
B. There are four sides.
C. The diagonals have the same length.
D. The opposite sides have the same length.
E. All of (A)-(D) are true in every rectangle.

8. A rhombus is a 4-sided figure with all sides of the same length.

Here are three examples.

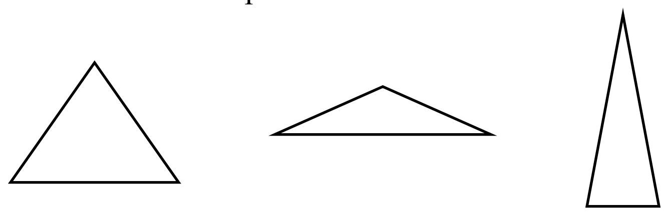


Which of (A)-(D) is not true in every rhombus?

A. The two diagonals have the same length.
B. Each diagonal bisects two angles of the rhombus.
C. The two diagonals are perpendicular.
D. The opposite angles have the same measure.
E. All of (A)-(D) are true in every rhombus.

9. An isosceles triangle is a triangle with two sides of equal length.

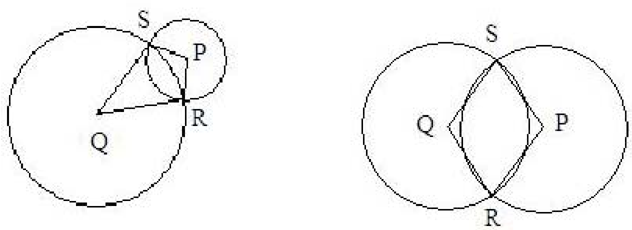
Here are three examples.



Which of (A)-(D) is true in every isosceles triangle?

A. The three sides must have the same length.
B. One side must have twice the length of another side.
C. There must be at least two angles with the same measure.
D. The three angles must have the same measure.
E. None of (A)-(D) is true in every isosceles triangle.

10. Two circles with centers P and Q intersect at R and S to form a 4-sided figure PRQS. Here are two examples.



Which of (A)-(D) is not always true?

A. PRQS will have two pairs of sides of equal length.
B. PRQS will have at least two angles of equal measure.
C. The lines $\overline{PQ}$ and $\overline{RS}$ will be perpendicular.
D. Angles P and Q will have the same measure.
E. All of (A)-(D) are true.

11. Here are two statements.

Statement 1: Figure F is a rectangle.
Statement 2: Figure F is a triangle.

Which is correct?

A. If 1 is true, then 2 is true.
B. If 1 is false, then 2 is true.
C. 1 and 2 cannot both be true.
D. 1 and 2 cannot both be false.
E. None of (A)-(D) is correct.

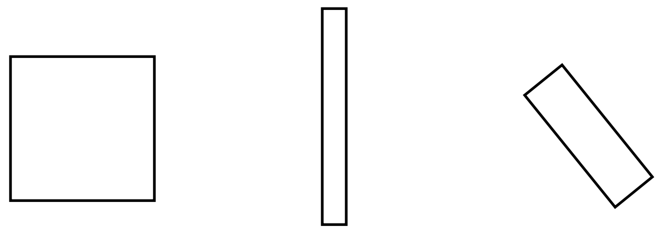
12. Here are two statements.

Statement S: ΔABC has three sides of the same length
Statement T: In ΔABC, $\angle B$ and $\angle C$ have the same measure.

Which is correct?

A. Statement S and T cannot both be true.
B. If S is true, then T is true.
C. If T is true, then S is true.
D. If S is false, then T is false.
E. None of (A)-(D) is correct.

13. Which of these can be called rectangles?



P Q R

A. All can.
B. Q only
C. R only
D. P and Q only
E. Q and R only

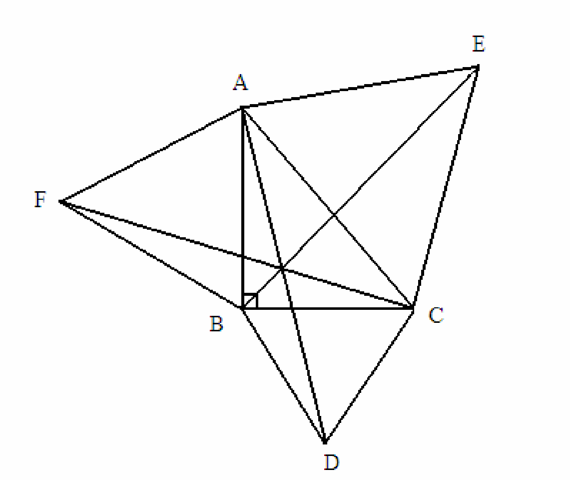
14. Which is true?

A. All properties of rectangles are properties of all squares.
B. All properties of squares are properties of rectangles.
C. All properties of rectangles are properties of all parallelograms.
D. All properties of squares are properties of all parallelograms.
E. None of (A)-(D) is true.

15. What do all rectangles have that some parallelograms do not have?

A. Opposite sides equal
B. Diagonals equal
C. Opposite sides parallel
D. Opposite angles equal
E. None of (A)-(D)

16. Here is a right triangle ABC. Equilateral triangles ACE, ABF, and BCD have been constructed on the sides of ABC.



From this information, one can prove that AD, BE, and CF have a point in common. What would this proof tell you?

A. Only in this triangle drawn can we be sure that $\overline{AD}$, $\overline{BE}$ and $\overline{CF}$ have a point in common.

B. In some but not all right triangles, $\overline{AD}$, $\overline{BE}$ and $\overline{CF}$ have a point in common.

C. In any right triangle, $\overline{AD}$, $\overline{BE}$ and $\overline{CF}$ have a point in common.

D. In any triangle, $\overline{AD}$, $\overline{BE}$ and $\overline{CF}$ have a point in common.

E. In any equilateral triangle, $\overline{AD}$, $\overline{BE}$ and $\overline{CF}$ have a point in common.

17. Here are three properties of a figure.

Property D: It has diagonals of equal length.
Property S: It is a square.
Property R: It is a rectangle.

Which is true?

A. D implies S which implies R.
B. D implies R which implies S.
C. S implies R which implies D.
D. R implies D which implies S.
E. R implies S which implies D.

18. Here are two statements.

I: If a figure is a rectangle, then its diagonals bisect each other.
II: If the diagonals of a figure bisect each other, the figure is a rectangle.

Which is correct?

A. To prove I is true, it is enough to prove that II is true.
B. To prove II is true, it is enough to prove that I is true.
C. To prove II is true, it is enough to find one rectangle whose diagonal bisect each other.
D. To prove II is false, it is enough to find one non-rectangle whose diagonals bisect each other.
E. None of (A)-(D) is correct.

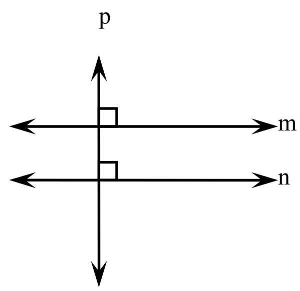
19. In geometry:

A. Every term can be defined and every true statement can be proved true.
B. Every term can be defined but it is necessary to assume that certain statements are true.
C. Some terms must be left undefined but every true statement can be proved true.
D. Some terms must be left undefined and it is necessary to have some statements which are assumed true.
E. None of (A)-(D) is correct.

20. Examine these three sentences.

1. Two lines perpendicular to the same line are parallel.
2. A line that is perpendicular to one of two parallel lines is perpendicular to the other
3. If two lines are equidistant, then they are parallel.

In the figure below, it is given that lines m and p are perpendicular and lines n and p are perpendicular. Which of the above sentences could be the reason that line m is parallel to line n?



A. (1) only
B. (2) only
C. (3) only
D. Either (1) or (2)
E. Either (2) or (3)

Appendix C

End-of-Semester Geometry Survey

End-of-Semester Geometry Survey   Name: ____________

Key: 5 = strongly agree; 4 = agree; 3 = no opinion; 2 = disagree; 1 = strongly disagree

1. Overall, I enjoy school.	5	4	3	2	1
2. Last year, math made sense to me.	5	4	3	2	1
3. This year, Geometry is making sense to me.	5	4	3	2	1
4. I believe it is possible to make sense of math.	5	4	3	2	1
5. Math is a useful subject to learn.	5	4	3	2	1
6. Learning Geometry by working on new problems and discussing student solutions helps me make sense of it.	5	4	3	2	1
7. Learning Geometry by listening to a lecture and taking notes helps me make sense of it.	5	4	3	2	1

8. If you disagree with statements 6 & 7, explain how you would best learn Geometry:

9. In 1 or 2 sentences, describe how we have been learning Geometry this year.

==

Key: 4 = often; 3 = some; 2 = little; 1 = not at all

10. I complete my homework assignments on time.	4	3	2	1
11. I study for my quizzes and tests.	4	3	2	1
12. I ask questions in class when I am confused.	4	3	2	1
13. I get extra help after school or with a tutor when I am still confused.	4	3	2	1

14. Are you pleased with your performance and/or grade during the 1st semester? Why or why not?

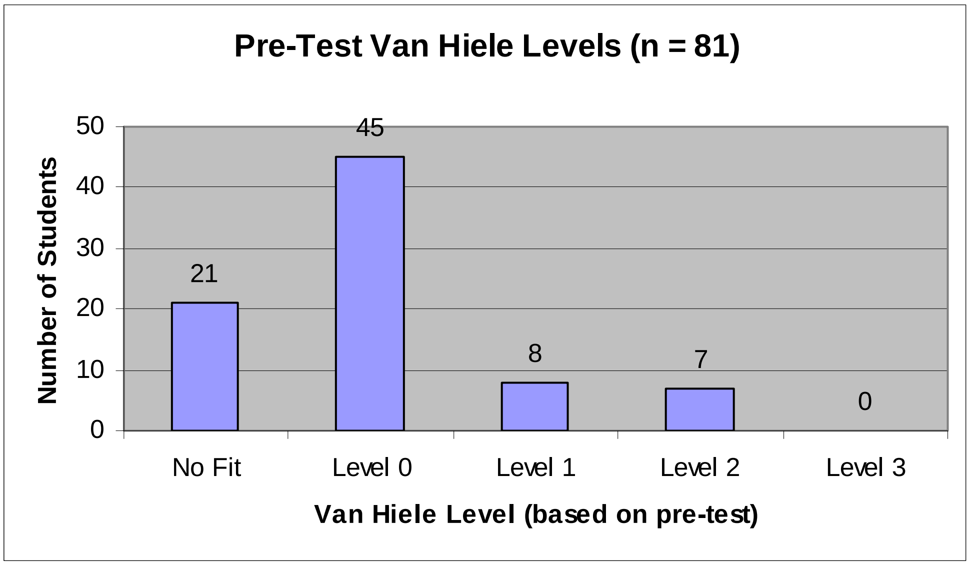
15. What can you do differently during the 2nd semester to be more successful in Geometry? *Explain why it would help you.*

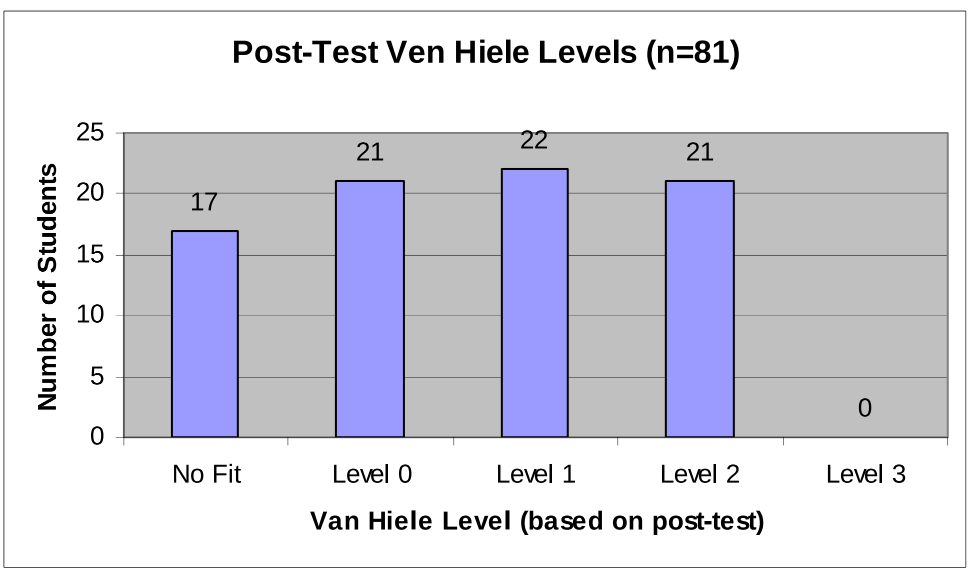
16. What can Mr. Henry do differently during the 2nd semester to help you be more successful in Geometry? *Explain why it would help you.*

Appendix D

van Hiele Geometry Test Results

(Note: Only 81 of the 87 students were present to take both the pre-test and the post-test)





Appendix E

End-of-Semester Geometry Survey Results

(Note: Only 79 of the 87 students were present to take the survey)

Question	Student Responses 5	4	3	2	1	Avg.
1. Overall, I enjoy school.	6	29	30	9	5	3.28
2. Last year, math made sense to me.	13	23	17	15	11	3.15
3. This year, Geometry is making sense to me.	16	26	20	15	2	3.49
4. I believe it is possible to make sense of math.	30	32	12	4	1	4.09
5. Math is a useful subject to learn.	24	34	16	4	1	3.96
6. Learning Geometry by working on new problems and discussing student solutions helps me make sense of it.	12	34	27	6	0	3.66
7. Learning Geometry by listening to a lecture and taking notes helps me make sense of it.	7	17	26	22	7	2.94
10. I complete my homework assignments on time.	na	38	29	9	3	3.29
11. I study for my quizzes and tests.	na	14	39	16	10	2.72
12. I ask questions in class when I am confused.	na	23	28	22	6	2.86
13. I get extra help after school or with a tutor when I am still confused.	na	16	17	23	23	2.33

Key for Questions 1-7: 5 = strongly agree; 4 = agree; 3 = no opinion; 2 = disagree; 1 = strongly disagree

Key for Questions 10-13: 4 = often; 3 = some; 2 = little; 1 = not at all

Questions 8, 9, 14, 15 and 16 were all free-response questions and are not included in these statistical results.